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II. *The Lunar Diurnal Magnetic Variation at Greenwich and other Observatories.*

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PART IV.—SUMMARY AND THEORETICAL DISCUSSION OF THE MAIN FEATURES OF L.

Introduction.

§ 1. The present research forms part of a wider investigation of terrestrial magnetism, the main object of which is the study of certain electrical phenomena that are associated with solar emissions absorbed in the upper atmosphere, and with the systematic motions of the upper atmosphere. The subject also bears on the electrical conductivity of the solid earth and oceans. The results are briefly discussed from this standpoint in Part IV.

The immediate subject of the paper is the lunar diurnal variation of the earth's magnetic field,* and particularly that of the declination at Greenwich, although the results of extensive reductions for other elements, at Batavia, Zikawei, and Pavlovsk, are also included.

In earlier papers† dealing with L, my material consisted of seven years' hourly observations of the three magnetic elements at five observatories, namely, Pavlovsk, Pola, Zikawei, Manila and Batavia. The original object of these reductions was to ascertain the chief spherical harmonic terms in the potential function representing the magnetic field of L, but the material was also used for the discussion of the dependence of L on the moon's distance and on the epoch in the solar cycle. In determining the potential function it was as important to find in what circumstances L is small, as to obtain its value accurately when comparatively large. This having been done, however, it seemed advisable, when I decided to examine further the dependence of L on various factors, to select for treatment, at least in the first instance, only those elements at those seasons and stations, for which the amplitude of L is sufficiently large to afford hope of detecting moderate percentage changes in it.

The purpose of this extended investigation was to examine in more detail how L is affected by changes in the moon's distance and in the sunspot epoch. But the influence of the magnetic activity, which varies from day to day even at the same sunspot epoch, was also examined, though it was expected to be negligible or small. This anticipation was not confirmed, however, and the amplitude of L proved to depend more upon the magnetic activity than on any other factor. The scope of the investigation, which was

* For the sake of brevity the lunar diurnal magnetic variation will throughout this paper be referred to by the symbol L, and the solar diurnal magnetic variation will similarly be denoted by S. The suffix 1, 2, 3, or 4 attached to L or S will indicate that the corresponding harmonic component of the lunar or solar diurnal magnetic variation is referred to.

† 'Phil. Trans.,' A, vol. 218, p. 1, 1917; also vol. 213, p. 279, 1913; vol. 214, p. 295, 1914; vol. 215, p. 161, 1915.

at first confined to the magnetic declination at Batavia, was thereupon extended to include other magnetic elements at other observatories also (Zikawei and Pavlovsk). In each case use was made of all the available data for the four (Northern or Southern) summer months, during which the amplitude of L is greatest. This work has been in progress since the year 1918.

In 1920, as a result of discussions with the Astronomer Royal, the Greenwich investigation, which forms the main part of this paper, was commenced. The Greenwich series of hourly values of the magnetic elements is among the longest extant, and has been extensively studied from many standpoints. It has not, however, hitherto been reduced by the method here used for the investigation of L. The present Greenwich research, which is confined to the magnetic declination, forms the most comprehensive investigation of L yet made for any single observatory and element. Its execution was rendered possible by the kind co-operation of the Astronomer Royal, whose interest and generous assistance I wish gratefully to acknowledge.

The Greenwich data were reduced under the writer's supervision, partly at the Royal Observatory, by members of the observatory staff, and partly elsewhere, by Mr. W. S. FRANKS, who also did a large part of the reduction of the data from Batavia, Zikawei and Pavlovsk. Miss D. WITHINGTON, M.Sc., of Manchester University, did the whole of the reductions for the horizontal magnetic force at Batavia, and gave valuable help in other ways. Messrs. ACTON, CULLEN, EDNEY, TURNER and WITCHELL, of the Royal Observatory, Greenwich, have all taken a considerable share in the computations involved in this paper.

Part of the Greenwich investigation was executed at the cost of the Royal Observatory, while the remainder of the expenses involved in the Greenwich and other reductions was borne by a grant placed at the writer's disposal by the Government Grant Committee of the Royal Society.

The method of reduction used in this paper (§§ 3–10, especially §§ 9, 10) is novel, and was developed in 1918 in connection with the work on the magnetic declination at Batavia.

PART I.—THE GREENWICH DATA AND THEIR REDUCTION.

The declination data.

§ 2. Hourly eye observations of magnetic declination began at Greenwich in 1840, and were made till 1847 in the upper room of the old magnetic observatory. In the latter year photographic registration was introduced. In 1864 the magnets were removed to the then new Magnetic Basement. The photographic sheets for the years 1864–67 were not measured or reduced, partly owing to frequent experimental changes of suspension. Hourly measures (at the exact hours) have been made from the subsequent records, and the mean daily values and monthly mean hourly values thence derived (with the aid of suitable absolute measurements) have been printed in the annual volumes of the Observatory. In 1914 the old declination magnet was superseded by a new instrument housed in a newly built observatory, and since **1916** hourly values have been printed in the volumes of Greenwich Observations. The hourly values up to **1914** were not published. It was decided to confine the present investigation to the results from the old instrument, covering the period of 63 years, **1848–63**, **1868–1914**.

The number of lunar days on which sufficiently complete observations were available for this investigation was 20,762, an average of 330 per year. The number of lunar days per year is approximately 350; the difference of 20 days per year is due to days of lost or incomplete record, either during periods of great disturbance or, especially in the earlier years, periods when the instrument was under adjustment or repair. The number of lunar daily sequences for each individual year is given in Table I, p. 55.

The hourly values derived from the declination magnet were recorded in arc to one-tenth of a minute ($0' \cdot 1$). The declination was westerly at Greenwich throughout the whole period, and this has therefore been adopted as the positive direction. The value of $1'$ change in declination, expressed in force units, was $5 \cdot 27\ \gamma$ in the mean for the whole period.

The general Plan of the Reductions.

§ 3. The determination of L involves the re-arrangement of the hourly observations according to lunar time. But as S, the solar diurnal variation, is of nearly the same period and of much greater amplitude, it is desirable to remove this from the observations before re-tabulating them. This was done by subtracting from each hourly value the mean value at the same solar hour during the current month. A constant was added at the same time, so that the hourly differences should be positive. These hourly differences were the quantities re-tabulated on what will be referred to as the " lunar sheets."

The mean lunar day is of length 24 hours 50 minutes in mean solar time, so that 25 consecutive solar hourly observations (or hourly differences) will approximately represent the course of the declination during a lunar day. The hourly differences were actually re-entered on the lunar sheets in rows of 26, however, in order that the non-periodic part of the daily change in declination, due to annual or secular variation, might afterwards be removed from the lunar hourly inequalities. In any such " lunar sequence " of 26 entries, the first entry was for the solar hour nearest to the time of upper lunar transit, the lunar day being here reckoned from one upper transit to the next. This solar hour will be referred to as the initial solar hour. Usually the initial solar hours for successive lunar days differed by unity, and then the 26th entry for any given lunar day recurred as the first entry for the following day. Occasionally, however, the initial solar hours for consecutive lunar days would agree, in which case the last *two* entries for the earlier day recurred as the first two for the later day.

All the lunar sequences commencing in a given calendar month (without distinction of year) were entered on one set of 24 lunar sheets, which were headed 0h., 1h., ... 23h.; on each such sheet all the lunar sequences would have the same initial solar hour, that, namely, indicated at the head of the sheet. On any one lunar sheet the daily sequences were written in order of date. The year and day of the month on which the lunar day began were noted at the left of the row. The total number of sequences being 20,762, and there being 12 sets each of 24 sheets, the average number of lunar daily sequences on each sheet was approximately 70. Days of incomplete record were omitted unless not more than six hourly entries were missing; in the latter case the gaps were filled up by interpolation. Disturbed days were omitted only in cases where, on account of the rapidity of the oscillations, the hourly measurements had not been taken; the disturbed curves had to some extent been smoothed by freehand drawing before the hourly values were read off, a procedure which, when restricted within suitably narrow limits, is of advantage for the present use of the observations.

The entry of the lunar sequences on separate sheets, according to their initial solar hours, groups together days at the same lunar age or phase; it was done because of the important changes which L undergoes in the course of the synodic month (§ 10). The initial solar hours were reckoned in astronomical time (noon being 0h.), and since they referred to the hour of upper lunar transit, the days entered on the sheets headed 0h. began at new moon, those on the sheets headed 6h. at first-quarter phase, those on the 12h. sheets at full moon, and so on.

The hourly differences were entered to the nearest minute of arc, since previous investigations had shown that in averaging such data as these, for large numbers of days, the addition of the decimals of a minute would not add appreciably to the accuracy of the result.

Classification according to Lunar Distance.

§ 4. In order that it might be possible to examine how L varies with lunar distance, the daily sequences were grouped into four classes, denoted by the letters A, *a*, *p*, P (one of which was written at the left of each sequence). The days marked A or P included the actual days of apogee or perigee, together with the two preceding and two succeeding days. The remaining days were marked *a* or *p*, according as the corresponding lunar distances were greater or less than the mean distance.

Classification according to Daily Range, or Magnetic Activity.

§ 5. The range of the entries in each lunar sequence was noted at the right of the row. The range is essentially a measure of the irregular part of the magnetic variation, S having been removed before the retabulation (the part of the range due to L itself is usually less than one unit—1′—of the entries, and the same applies also to the seasonal and secular parts of the daily non-periodic change). The range was therefore adopted as a measure of the magnetic activity or disturbance. The results obtained by this method of classification indicate that it corresponds to an important systematic change in magnetic conditions, of which the daily range, obtained in the manner described, is a satisfactory rough measure.

It has for some years been the practice at many observatories to publish for each month not only the mean solar diurnal variation S as derived from the whole of each calendar month, but also that derived from the five magnetically quietest days. These days, which extend from one *Greenwich* midnight to the next, independent of the local time at the various observatories, are chosen in accordance with an international scheme by which, at each co-operating observatory, a "character figure" 0, 1, or 2 is assigned to each day; 0 denotes a quiet, 1 an intermediate, and 2 a disturbed day. The average character figures from all the observatories are used in choosing the five quietest days.

The practice is also gaining ground of giving, in addition, the mean solar diurnal variation derived from the five internationally most disturbed days in each month. Interesting differences are found to exist between S as determined from all days, and from the five quietest or most disturbed days in each month.

The writer's discovery, in 1918, that L also changes with the magnetic activity, made it desirable to classify the lunar days according to magnetic activity in such a way as to correspond as nearly as possible with the plan adopted for the solar diurnal variation: namely, by grouping together the five quietest and the five most disturbed days per month, so as to facilitate comparison between the corresponding changes in S and L. It is not practicable, on account of the overlapping of the lunar and solar days, to use the international character figures for classifying the lunar sequences, but there can be little doubt that the classification by daily range (after S has been removed) is a substantially equivalent method. But in order to choose limits of range which should give corresponding fractions (about one-sixth) of the whole number of days in the quiet-day and disturbed-day groups, it was first necessary to determine the frequency of occurrence of days on which the range was 1, 2, 3, ... units.

A "frequency table" was therefore constructed giving the number of days of each range in each separate calendar month of each year throughout the series, but without distinction between lunar days commencing at different solar hours. This table was then summarized so as to give monthly totals for five groups of years classified according to their mean sunspottedness (§ 6), and these totals were further combined into complete totals for each of these five groups of years (taking all calendar months together), and into twelve monthly and three seasonal totals for all the years taken together. In this and every other seasonal grouping made in this paper, (Northern) summer was represented by the four months May to August, winter by the four months November to February, and the equinoctial season by the four intervening months.

Before discussing the summary of the frequency table, and describing the classification according to magnetic activity which was based on it, it is convenient to indicate how the years were grouped together according to the average sunspottedness, or epoch in the solar cycle.

Classification of Years according to the Solar Epoch.

§ 6. The solar diurnal magnetic variation is found to depend materially on the epoch in the solar sunspot cycle; for example, at Greenwich, S is sometimes* nearly twice as large at sunspot maximum as at sunspot minimum (*cf.* § 21). No such marked change is shown by L, on the other hand, and the question whether or not L suffers *any* systematic

* It must be remembered that, as Table I shows, sunspot maxima differ considerably among themselves in their intensity.

change with the solar epoch has remained without a decisive answer. The long series of Greenwich records seemed to offer an unusually good opportunity of settling the point.

The 63 years covered by the material used were grouped according to their annual mean sunspot number, using the unsmoothed values given by WOLF and WOLFER (*cf.* Table I). Five nearly equal groups of years were formed, corresponding to the following limits of range in sunspot number; the groups, in descending order of sunspottedness, were denoted by the letters α, β, γ, δ, ε, and alongside each daily sequence on the lunar sheets was written the group-letter appropriate to the year to which the sequence belonged.

Group :—	α	β	γ	δ	ε
Range of sunspot number	>70	55–70	31–54	30–11	<11
Number of years in group	12	11	14	12	14

In the following table the sunspot number, the group letter, and the number of available lunar sequences, are given for each of the 63 years. The letters A and D, attached to certain years of the ascending or descending phases in the sunspot cycles, will be referred to later (§ 7).

TABLE I.

Year.	Mean sunspot number.	Group.	No. of lunar sequences.	Year.	Mean sunspot number.	Group.	No. of lunar sequences.	Year.	Mean sunspot number.	Group.	No. of lunar sequences.
				1870	139	α	312	1895	64	β D	352
				1	111	α D	327	6	42	γ D	354
				2	102	α D	336	7	26	δ D	345
1848	124	α	279	3	66	β D	345	8	27	δ D	345
9	96	α D	285	4	45	γ D	335	9	12	δ D	342
1850	66	β D	327	1875	17	δ D	347	1900	10	ε D	354
1	64	β D	324	6	11	δ D	333	1	3	ε	339
2	54	γ D	303	7	12	δ	348	2	5	ε	343
3	39	γ D	290	8	3	ε	346	3	24	δ A	351
4	21	δ D	306	9	6	ε	346	4	42	γ A	353
1855	7	ε	287	1880	32	γ A	342	1905	64	β	345
6	4	ε	262	1	54	γ A	324	6	54	γ	338
7	23	δ A	303	2	60	β A	332	7	62	β	357
8	55	β A	327	3	64	β	326	8	48	γ D	337
9	94	α A	309	4	64	β	348	9	44	γ D	347
1860	96	α	262	1885	52	γ D	329	1910	19	δ D	348
1	77	α D	307	6	25	δ D	338	1	6	ε	352
2	59	β D	287	7	13	δ D	344	2	4	ε	358
3	44	γ	343	8	7	ε	340	3	1	ε	348
4	47			9	6	ε	351	4	10	ε	348
1865	30			1890	7	ε	341				
6	16			1	36	γ A	327				
7	7			2	73	α A	304				
8	37	γ A	300	3	85	α	342				
9	74	α A	309	4	78	α D	333				

The Frequency Table of Daily Ranges.

§ 7. The frequency of occurrence of a given daily range among any group of lunar days is best expressed as a percentage of the whole number of days in the group. This is the plan followed in Table II, which is the frequency table for the several groups of days already described in § 5, together with two groups not there mentioned. The latter refer to days occurring in the years of ascending or descending phase in the solar cycle (or of increasing or diminishing sunspottedness), indicated by A or D in Table I; these groups of years were chosen so that the mean sunspottedness should be nearly equal in the two groups, viz.,

Mean sunspot number for 13 years (D) of decreasing sunspottedness : 47·2
,, ,, ,, ,, 27 ,, (A) of increasing sunspottedness : 47·7

No lunar sequence throughout the series was found to have zero range, *i.e.*, a range less than one unit (1′ or $5\cdot3\gamma$). The greatest range noted was 34′, though larger ranges must have occurred among the highly disturbed days which were omitted from the tabulations because the oscillations shown by the magnetograph could not be satisfactorily smoothed. This smoothing of the records must have had a general tendency to reduce the recorded ranges below the true values corresponding to the exact hourly readings though probably without altering the general relative order of days of different ranges. The smoothing is probably advantageous for the present purpose, though, on account of its element of arbitrariness, it is less desirable than the practice (adopted within recent years at Potsdam, Greenwich, and elsewhere) of tabulating average values over successive hourly periods instead of instantaneous values at exact hours. But on ordinary days, of course, there is little to choose between the different methods, and the great majority of hourly values used in the present investigation must have been measured directly from the unsmoothed curve.

Apart from its use in the classification of days according to their magnetic activity (§§ 6, 8), the frequency table (Table II) has an independent interest. So far as the writer is aware, such a table, giving the range freed from the solar diurnal magnetic variation, has not hitherto been published. Part of Table II is represented graphically in Figs. 1, 2, and 3. Fig. 1 illustrates the first part of Table II, and shows how the most frequent range, and the frequency of days of large range, increase with increasing sunspottedness; the most frequent range varies from 3′ to 5′. Fig. 2 illustrates the last part of Table II, showing that the winter season is distinctly less disturbed than the other two seasons: the most frequent range in winter is 3′, while in all the other months (excepting April) it is 4′; the spread of the larger ranges is much greater in the equinoctial months than in summer or winter. It is, of course, well known that magnetic disturbance is most common in the equinoctial months.

Fig. 3 illustrates columns A and D of Table II. This division of the data was made in order to see whether the solar causes which affect terrestrial magnetic conditions, such as the daily range, depend only on the degree of sunspottedness, or whether, for equal degrees of the latter, a difference in the magnetic conditions is traceable between the ascending and descending phases of the solar cycle. The latter seemed a possibility, in view of the changes in the mean latitudes of sunspots at different solar epochs ; but

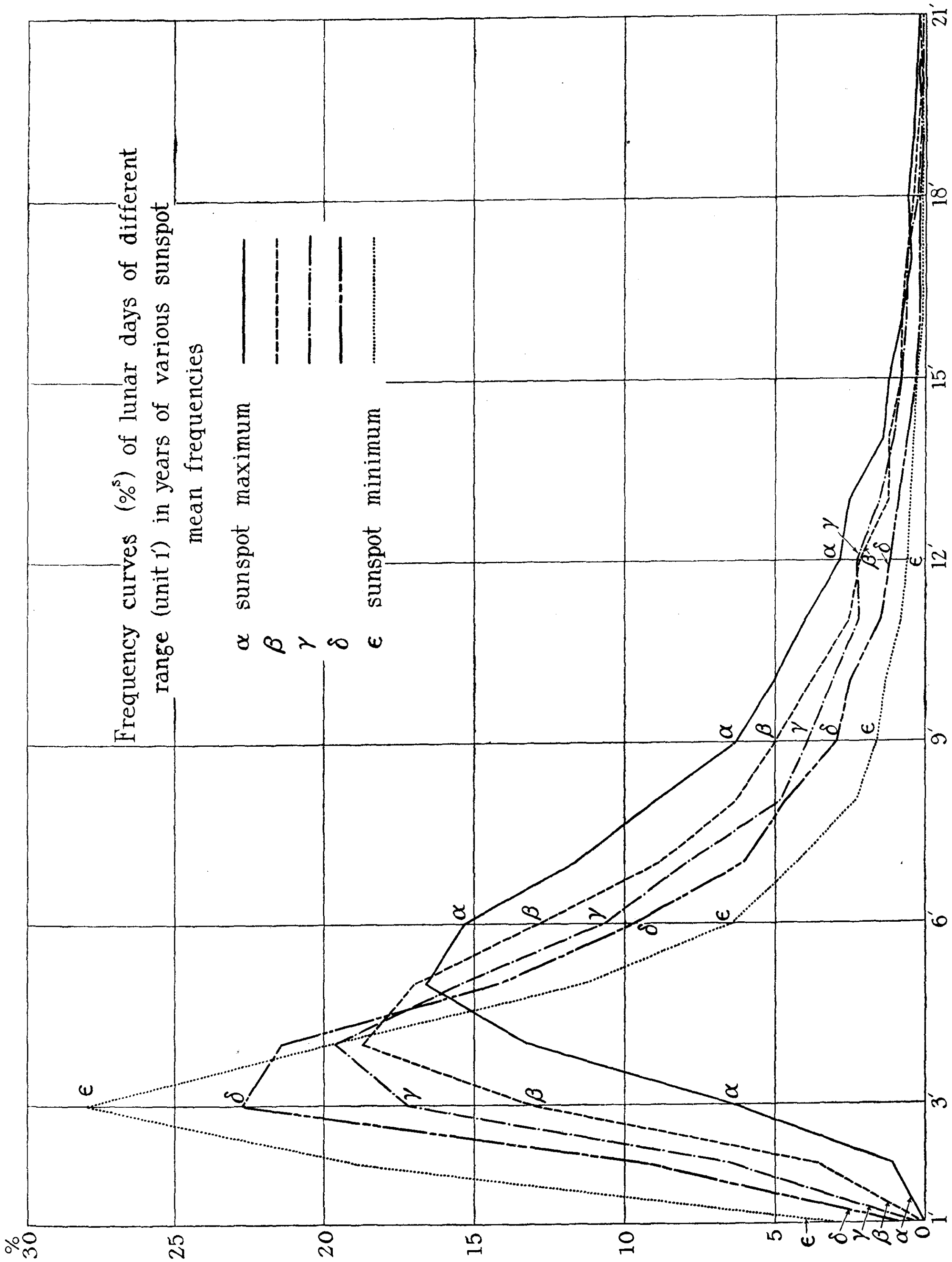


Fig. 1.

fig. 3 shows that there is no appreciable change, due to this cause, in the distribution of the daily ranges.

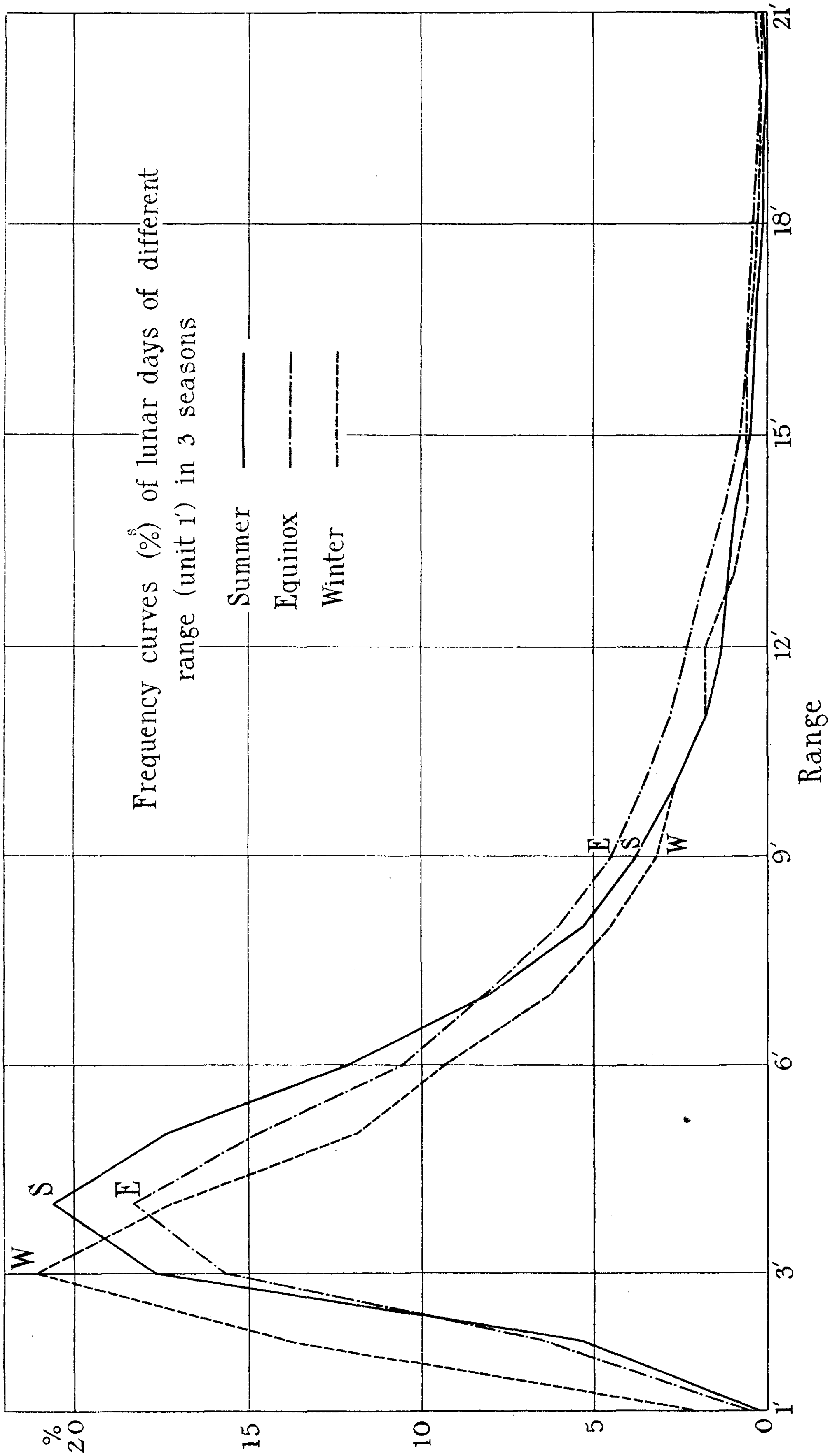
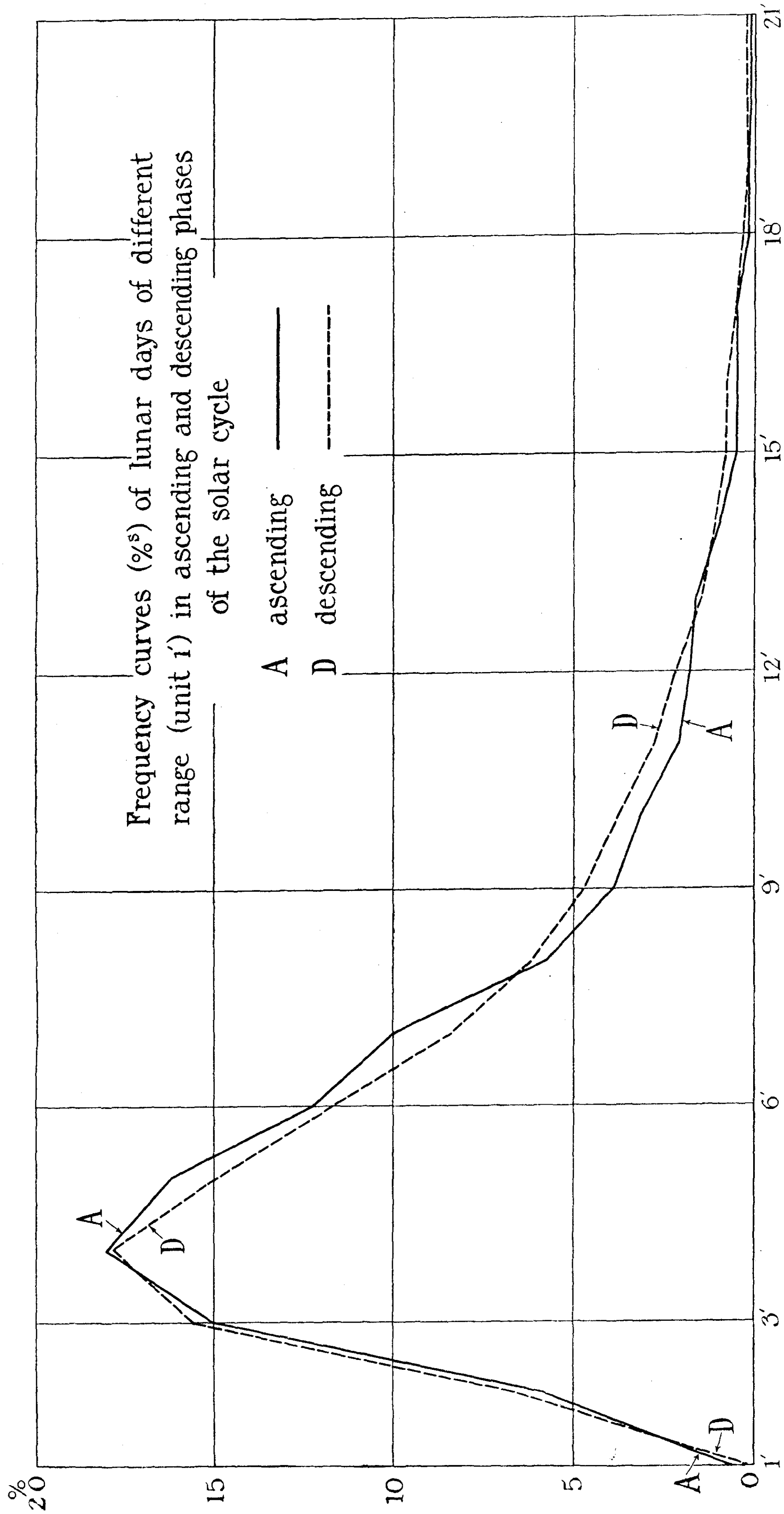

Frequency curves (%^{s}) of lunar days of different range (unit 1′) in 3 seasons
Summer
Equinox
Winter
%
20
15
10
5
0
1′
3′
6′
9′
12′
15′
18′
21′
W
S
E
Range

Fig. 2.


Frequency curves (%s) of lunar days of different range (unit 1′) in ascending and descending phases of the solar cycle
A ascending
D descending
%
20
15
10
5
0
1′
3′
6′
9′
12′
15′
18′
21′
A
D

Range

Fig. 3.

TABLE II.—Percentage numbers of lunar days of different range of Greenwich declination (after removal of the solar diurnal variation).

Range in minutes of arc.	Sunspot groups of years. α (max.).	β	γ	δ	ϵ (min.).	A.	D.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Summer.	Equinox.	Winter.
1	0·0	0·1	0·4	0·8	3·0	0·8	0·3	2·5	1·3	0·3	0·3	0·1	0·3	0·2	0·3	0·6	0·8	1·2	3·5	0·2	0·5	2·1
2	1·1	3·7	6·5	9·4	18·8	5·8	6·5	14·2	9·1	5·6	6·3	4·2	6·4	5·1	5·6	5·3	7·9	13·4	17·8	5·3	6·3	13·7
3	6·5	13·2	17·3	22·7	28·1	15·0	15·6	21·0	18·3	14·1	16·9	17·3	18·2	18·4	17·1	15·2	16·5	21·6	23·3	17·7	15·7	21·1
4	13·2	18·7	19·6	21·5	19·9	18·0	17·9	17·9	16·8	18·6	16·9	19·5	21·6	21·1	20·3	19·4	18·4	17·6	16·5	20·6	18·3	17·2
5	16·6	16·9	15·5	14·1	11·3	16·2	14·9	11·0	14·0	15·1	15·5	18·1	17·2	17·1	17·1	15·8	13·0	12·3	10·5	17·4	14·8	11·9
6	15·3	12·8	10·6	9·6	6·4	12·2	11·7	8·7	11·4	11·4	10·5	12·1	13·5	11·2	11·7	10·3	9·9	9·4	7·9	12·1	10·5	9·3
7	11·6	8·8	7·9	6·0	4·2	10·0	8·4	6·1	7·5	8·8	8·9	8·5	7·7	8·4	7·7	7·8	7·4	6·3	5·3	8·1	8·2	6·3
8	9·0	6·3	4·8	4·7	2·3	5·8	6·2	4·7	5·0	6·4	7·0	5·7	5·2	5·3	4·9	4·8	5·6	4·4	3·8	5·3	6·0	4·5
9	6·3	5·0	3·9	2·9	1·6	3·9	4·7	3·1	3·7	4·5	4·2	3·7	2·6	4·0	4·8	4·8	4·4	3·4	2·6	3·8	4·5	3·2
10	5·0	3·8	3·1	2·5	1·3	3·2	3·8	2·5	3·0	2·8	3·1	3·0	2·5	2·6	2·8	4·9	3·8	3·1	2·2	2·7	3·6	2·7
11	4·0	2·5	2·2	1·5	0·8	2·1	2·8	1·7	1·9	3·5	2·4	1·9	1·1	2·1	2·3	2·5	2·6	2·0	1·6	1·8	2·8	1·8
12	2·8	2·2	2·3	1·2	0·6	1·8	2·2	2·0	2·3	2·8	1·9	1·5	1·3	0·8	1·5	2·0	2·4	1·4	1·4	1·3	2·3	1·8
13	2·5	1·2	1·5	0·9	0·5	1·6	1·5	0·9	1·4	1·9	1·3	1·6	0·9	1·0	1·1	2·0	1·9	0·9	0·8	1·1	1·8	1·0
14	1·4	1·2	1·1	0·6	0·4	1·0	1·1	0·8	0·8	1·4	0·8	0·9	0·5	1·0	1·2	1·6	1·2	0·6	0·3	0·9	1·2	0·6
15	1·2	0·8	0·8	0·3	0·3	0·5	0·8	0·7	0·7	0·8	0·8	0·5	0·4	0·6	0·6	0·8	0·9	0·6	0·3	0·5	0·8	0·6
16	0·8	0·8	0·7	0·2	0·1	0·5	0·8	0·3	0·9	0·3	0·9	0·2	0·2	0·4	0·6	0·5	0·7	0·5	0·7	0·4	0·6	0·6
17	0·5	0·6	0·6	0·2	0·1	0·5	0·5	0·6	0·4	0·5	0·6	0·4	0·3	0·3	0·1	0·3	0·5	0·3	0·2	0·3	0·5	0·4
18	0·6	0·4	0·3	0·2	0·1	0·2	0·3	0·5	0·4	0·2	0·3	0·2	0·0	0·0	0·1	0·4	0·6	0·2	0·2	0·1	0·4	0·3
19	0·4	0·3	0·2	0·1	0·1	0·2	0·2	0·6	0·0	0·0	0·1	0·1	0·0	0·1		0·4	0·5	0·3	0·1	0·1	0·3	0·2
20	0·3	0·1	0·1	0·2	0·1	0·1	0·2	0·0	0·3	0·2	0·2	0·0	0·0	0·0		0·1	0·3	0·3	0·3	0·0	0·2	0·2
21	0·2	0·2	0·2	0·1	0·1	0·1	0·2	0·2	0·1	0·2	0·3	0·1	0·0	0·1		0·2	0·3	0·1	0·0	0·1	0·3	0·1
22	0·1	0·1	0·1	0·1		0·1	0·1		0·1	0·3	0·1	0·1	0·0	0·0		0·1	0·0	0·0	0·2	0·0	0·1	0·1
23	0·1	0·1	0·1	0·1			0·1		0·0	0·0	0·3	0·1	0·1	0·0		0·2	0·1	0·1	0·0	0·1	0·1	0·0
24	0·0	0·1	0·1				0·0		0·0	0·2		0·0		0·0			0·0		0·0	0·0	0·1	0·0
25	0·2	0·1	0·1				0·1		0·3			0·1		0·2			0·0		0·1	0·1		0·1
26	0·1																0·1					
27																						
>27																						

Limits of Range in the Activity-Groups of Days.

§ 8. In classifying the lunar daily sequences into quiet-day (Q), ordinary (O), disturbed-day (X) and very disturbed (XX) groups, according to the diurnal range, it seemed desirable to take account of the considerable seasonal variation (Table II) in the days of small range. Otherwise it would be impossible even approximately to make the Q groups contain about five out of each thirty daily sequences, as is done in the international scheme for determining S on quiet days (§ 5). The range-groupings actually made are given below, together with the proportion of days which they include reckoned as the average number per month of 30 days. In the last group, of specially disturbed days, no distinction of season was made, owing to the relatively small size of the group and the large accidental variations occurring on these days.

	Q (quiet).		O (ordinary).		X (disturbed).		XX (very disturbed).	
	Ranges.	Days per month.	Ranges.	Days per month.	Ranges.	Days per month.	Ranges.	Days per month.
Summer...	1′, 2′, 3′	7·0	4′–7′	17·4	8′–13′	4·8		
Equinox	1′, 2′, 3′	6·8	4′–7′	15·6	8′–13′	6·3	>13′	1·1
Winter .	1′, 2′	4·8	3′–7′	19·8	8′–13′	4·4		

Though this classification makes the group Q correspond only roughly with the five quietest days per month, no better grouping was possible without introducing a further decimal place (units of 0′·1) in all the tabulations, which would have increased the labour too greatly.

The Q group is not equally recruited from all years ; the years of small sunspot number contribute far more than those of large sunspot number. The converse is the case with regard to the groups X and XX. This difference of distribution of the Q and X days in different years is an inevitable consequence of a classification based on the daily range ; it is, moreover, not disadvantageous, since it appears that the lunar diurnal variation depends far more on the range than on the mean sunspot number (or the solar epoch). The subsequent work was arranged, however, so as to permit comparison between L (for days of equal range) in years of great and years of small sunspottedness.

The appropriate group letter Q, O, X, or XX was written alongside each daily sequence on the lunar sheets.

The Method of Summarising the Lunar Daily Inequalities.

§ 9. The various factors whose influence upon L it was desired to examine were (i) season, (ii) lunar distance, (iii) magnetic activity, and (iv) sunspottedness.

The influence of lunar age or phase was considered to be known (§ 10), but this also was examined briefly for confirmation or better illustration of previous knowledge (§ 17).

Totals were first formed for the 26 columns on each of the 24 lunar sheets for each calendar month. Subtotals of each column were also formed from the daily sequences falling within the five groups of years α, β, γ, δ, ε of different degrees of sunspottedness, and the sum of the subtotals was checked against the complete total for each column. The number of days involved in each total or subtotal was noted alongside each row of these sums.

The rows of complete totals for each of the 24 lunar sheets forming the set for any given calendar month were then re-written on a summary sheet, in order, beginning with the row of totals from the lunar sheet headed 0h., and ending with that for the sheet headed 23h. The number of days involved in each row of totals was noted alongside. There were twelve such summary sheets, one for each calendar month; on these sheets the data for all the 63 years were combined together.

Fifteen other summary sheets were formed by combining the subtotals for the months falling in each *season*—summer, equinox, or winter—but keeping separate the groups of years α, β, γ, δ, ε. On these and all other summary sheets there were 24 rows, corresponding to the different solar commencing hours 0h., 1h.; ... 23h.; and in every case the number of daily sequences included in the sums was indicated on the left of each row.

On each of the 12×24 lunar sheets nine other rows of subtotals were formed, as follows: the days in the activity-groups Q and X were divided according to sunspottedness only, each into two subgroups $Q\alpha\beta\gamma$, $Q\delta\varepsilon$, or $X\alpha\beta\gamma$, $X\delta\varepsilon$, including the sequences in the years falling in the sunspot groups indicated; the days in group XX were not subdivided at all; the days in group O were divided into four subgroups according to lunar distance, viz., OA, O*a*, O*p*, OP (§ 4). The sum of the nine subtotals ($Q\alpha\beta\gamma$, $Q\delta\varepsilon$, OA, O*a*, O*p*, OP, $X\alpha\beta\gamma$, $X\delta\varepsilon$, XX) on each lunar sheet was checked against the total sum, for each column.

These additional subtotals were combined on summary sheets as follows: one XX summary sheet, combining all calendar months (but keeping separate the 24 sets each corresponding to one solar commencing hour) was formed; these specially disturbed days were not numerous enough to admit of subdivision, particularly in view of the necessity for averaging out the considerable irregular changes occurring on them. The $Q\alpha\beta\gamma$, $Q\delta\varepsilon$, $X\alpha\beta\gamma$, $X\delta\varepsilon$ subtotals for all calendar months were likewise combined, giving four more summary sheets; in addition, the two Q subtotals ($Q\alpha\beta\gamma$, $Q\delta\varepsilon$) on each lunar sheet were added together, and further combined with other months, but keeping the seasons separate, so as to provide three seasonal summary sheets for days in group Q; three seasonal summary sheets for days in group X were similarly formed. Ten summary sheets were formed from the O subtotals, namely, six seasonal ones in which the groups OA and O*a*, or O*p* and OP, were combined, and four groups OA, O*a*, O*p*, OP, combining all months.

The total number of summary sheets was thus forty-eight. On each row of any summary sheet there was entered, in order, (i) the corresponding solar commencing hour, (ii) the number of sequences from which the row of sums was derived, (iii) 26 sums of lunar hourly differences, in columns headed 0h., 1h., 25h. The difference between the first and last of these individual sums in any row represented the total non-periodic variation of the declination on the corresponding group of days; this was removed by subtracting, from each entry after the first, a proportionate amount of the change. In this way the 26th entry (for 25h.) was reduced so as to equal the first entry (for 0h.). The revised rows of sums were entered on fresh summary sheets, and the modified 26th entries were then omitted.

The Summation of the Summary Sheets.

§ 10. It has already been mentioned, in § 3, that the separation of the lunar sequences according to their solar hour of commencement, which groups together days at the same lunar "age" or phase, was made because the lunar diurnal magnetic variation changes in a definite and important way with the age of the moon. The harmonic components of L (denoted by L_1, L_2, L_3, L_4...) are not altered in amplitude, but the phase of component r (L_r) regularly advances in the course of each lunation by $2(r-2)\pi$; only the second (semidiurnal) component L_2, therefore, remains independent of the moon's age, while the first or diurnal component L_1 retrogresses in phase by 2π per lunation, and the ter-diurnal and quarter-diurnal components L_3 and L_4 advance in phase by 2π and 4π respectively.

Owing to the constancy of the component L_2, the simple summation of the vertical columns on any summary sheet gives an inequality which contains this component to its full extent, and from which all the other components, owing to their cyclic changes of phase, disappear completely, provided the rows of sums for the different solar commencing hours are derived from equal, or approximately equal, numbers of similar days.

Such a row, of direct columnar sums, was formed on each summary sheet; it gave the component L_2 (and no other component) and was denoted as Sum II. The total number of days concerned in it was noted on the left.

To determine the other components it was necessary to allow, as nearly as possible, for their change of phase from one row to the next on the summary sheets. This was effected as follows.

Imagine the 24 rows (each containing 25 hourly entries) on any summary sheet to be written round a cylinder, so that the 1st column follows after the 25th just as the latter does after the 24th. The sums I, III, IV, designed to include the first, third and fourth harmonic components of L as fully as possible, were formed by processes which would correspond to simple vertical summations (or summations parallel to the axis) if successive rows, or groups of rows, were first rotated through certain angles round the cylinder, forward or backward, so as to alter the columns which come under the hourly entries on the first row. On the actual written summary sheets these processes represent various modes of "sloping summation," which are perhaps most simply described by using this illustration of the cylinder.

Sum I was obtained by a summation sloping forward and downward, equivalent to vertical summation on the cylinder after making a backward shift of the row of sums corresponding to the solar commencing hour n, through n columns for $n = 0, 1, 2, \ldots 11$, and through $n + 1$ columns for $n = 12, 13, \ldots 23$.

Sums III and IV were obtained by summations sloping backward and downward in *groups* of rows, equivalent to vertical summations on the cylinder after making forward shifts as follows: in the case of sum III, each triple set of rows for the solar commencing hours $3n - 1$, $3n$, $3n + 1$ (where n ranges from 1 to 7) would be shifted forward through n columns, the rows 23 h., 0 h., 1 h. remaining unmoved; in the case of sum IV, each pair of rows $2n^h$, $2n + 1^h$ ($n = 1, 2, \ldots 11$) would be shifted forward through n columns.

Since the interval between the moon's ages corresponding to two consecutive rows on the summary sheets is 1/24th of a lunation, the changes of phase from one row to the next in the components L_1, L_3, L_4 are respectively—15°, 15°, 30°. A backward or forward shift of any row through m columns is equivalent to an alteration of the initial epoch of the lunar day (supposed expressed in arc at the rate 2π or 360° per day) by $m/25$ of 360°, *i.e.*, by $m.\ 14^\circ\cdot4$. It is thus equivalent to an increase or decrease, respectively, of the phase of component L_r by r times this amount, *i.e.*, by $mr\ .\ 14^\circ\cdot4$. Let it be the summary row corresponding to the solar commencing hour p which is thus shifted, the shift being reckoned positive if backward (corresponding to a summation sloping forward-downwards). The phase of L_r in this row is, at the outset, $\phi_r + p\ (r - 2)\ 15^\circ$, where ϕ is the phase in row 0^h; after the shift, therefore, the phase is

$$\phi + p\ (r - 2)\ 15^\circ + mr.\ 14^\circ\cdot4.$$

The above methods of sloping summation are chosen so that this differs from ϕ by as small an amount as possible.

Consider, for example, the formation of Sum III. It will be supposed that the 24 rows on the summary sheet refer to substantially similar data, and that L_r is the same in each row except for the above-mentioned change of phase $(r - 2)\ 15^\circ$. In forming sum III the rows are taken in groups of three, having commencing hours $3n - 1$, $3n$, $3n + 1$; thus the components L_r in the first and third rows of such a group will differ in phase from L_r in the middle row by $\pm\ (r - 2)\ 15^\circ$. Hence the sum of the three rows will contain L_r with the same phase as in the middle row, but with amplitude diminished in the ratio $\frac{1}{3}\{1 + 2\cos(r - 2)\ 15^\circ\}$, which will be denoted by f_r. Clearly for $r = 3$, f_r is $0\cdot977$. Now in this summation the above group of rows is supposed shifted forward through n columns; the phase of L_3 in the sum for the group after the shift is consequently obtained by writing $p = 3n$, $m = -n$, $r = 3$ in the above formula, viz., it is

$$\phi_3 + 3n\ (3 - 2)\ 15^\circ - n.3.14^\circ\cdot4$$

or

$$\phi_3 + 3n\ (15^\circ - 14^\circ\cdot4)$$

or

$$\phi_3 + 1^\circ\cdot8\ n.$$

The shift of phase in L_3 from one group to the next is therefore nearly rectified for this component. Since n takes the values 1 to 7, the mean phase of L_3 in sum III will be

$$\phi_3 + \tfrac{1}{8}(0 + 1 + 2 + \ldots + 7)\, 1^\circ{\cdot}8,$$

that is,

$$\phi_3 + 6^\circ{\cdot}3.$$

By resolving along this mean phase, moreover, it is easy to see that the amplitude in the *sum* of the eight groups of three rows each is reduced, as compared with the amplitude in the sum of any one group, in the ratio

$$\tfrac{1}{8}(2 \cos 0^\circ{\cdot}9 + 2 \cos 2^\circ{\cdot}7 + 2 \cos 4^\circ{\cdot}5 + 2 \cos 6^\circ{\cdot}3)$$

or $0{\cdot}997$. The amplitude in the sum for any one group is already reduced in the ratio f_3 or $0{\cdot}977$; thus the total reduction of amplitude is $0{\cdot}997 \times 0{\cdot}977$ or $0{\cdot}974$, so that the amplitude of L_3 as derived by harmonic analysis of sum III (allowing for the number of lunar days involved in the sum) must be multiplied by the reciprocal of $0{\cdot}974$, that is, by $1{\cdot}026$). The phase of L_3 as derived by harmonic analysis of L_3 must be corrected by subtracting $6^\circ{\cdot}3 + 7^\circ{\cdot}5$ or $13^\circ{\cdot}8$ from it; the first part of this correction, $6^\circ{\cdot}3$, represents the correction to obtain the phase ϕ_3, in row 0 h.; the $7^\circ{\cdot}5$ reduces the latter phase to the phase of L_3 at the epoch of new moon, and is equal to the change of phase of L_3 during the half day between the *middle* and the *commencement* of the lunar day which *starts* at new moon (and which therefore has the solar commencing hour 0).

In a similar way it is found that L_1, as derived from sum I, requires correction to its amplitude by the factor $1{\cdot}003$, and to its phase by the addition of $- 0^\circ{\cdot}3 + 7^\circ{\cdot}5$, *i.e.*, $7^\circ{\cdot}2$; while L_4, derived from sum IV, requires corrections to amplitude by the factor $1{\cdot}046$, and to the phase by the addition of $- 28^\circ{\cdot}2 - 15^\circ$ or $- 43^\circ{\cdot}2$.

These corrections are summarised in the following table:

Component:	L_1.	L_2.	L_3.	L_4.
Amplitude-factor	1·003	1	1·026	1·046
Phase correction	7°·2	0°	−13°·8	−43°·2

The very slight differences between these amplitude-factors and unity are a measure of the success of the above methods of sloping summation in utilizing almost to the full the available information about L_1, L_3, and L_4 contained in the given data (previous to the writer's paper of 1913—*cf.* §1, footnote—these components had not been determined at all). This is due to the nearly exact equalization of the phases of L_r in the rows as actually summed in sum r; and the equalization has the advantage that small variations in the numbers of days corresponding to the various rows of sums on the summary sheets have very small influence on the final determination of L_r.

The Harmonic Analysis.

§ 11. The harmonic analysis of the various sets of sums, I, II, III, IV, was effected by the ordinary method appropriate to sequences of 25 values, using the formulæ

$$\sum_n (a_n \cos nt + b_n \sin nt)$$

and

$$\sum_n C_n \sin (nt + \theta_n),$$

and applying the above corrections to the amplitudes and phases in order that the phases might refer to the epoch of new moon. Each sum was analyzed only for the component which it was specially designed to give. The initial time in these formulæ was the time of upper transit of the moon at Greenwich.

The coefficients a_n, b_n, C_n were expressed in force units of $0 \cdot 01\ \gamma$ or 10^{-7} c.g.s. The positive direction of the force was westerly, normal to the magnetic north at Greenwich.

PART II.—DESCRIPTION AND REDUCTION OF THE DATA FROM BATAVIA, ZIKAWEI AND PAVLOVSK.

Description of the Data.

§ 12. The following table gives certain details of importance relative to the observational material used in this part of the investigation. The particulars for Greenwich are also included.

	Batavia.	Zikawei.	Pavlovsk.	Greenwich.
Latitude	6°·2 S.	31°·2 N.	59°·7 N.	—
Longitude	106°·8 E.	121°·4 E.	30°·5 E.	0°
Magnetic declination	0°·9 E.	2°·4 W.	0°·6 E.	—
Horizontal force	36690 γ	32860 γ	16550 γ	—
1′ of declination in force units	10·68 γ	9·56 γ	4·81 γ	5·27 γ
Calendar months dealt with	Nov.–Feb.	May–Aug.	May–Aug.	All.
Elements investigated	Declination, H. F.	Declination.	Declination, H.F., V.F.	Declination.
Approximate period* covered by the data	1882–1912	1877–1911	1870–1908	1848–1914

The method of reduction was in general the same as that described for Greenwich in Part I, though the data, being less numerous, were less minutely sub-divided.

§ 12.1. *Declination at Batavia.*—Up to 1899 February, the Batavia records gave east declination in arc measurement: subsequently east force, in γ, was recorded instead of the declination. The declination at Batavia is so small that the east force records were regarded in this investigation as equivalent to declination reckoned in force units.

* In each case there were gaps of one or more years in these periods, owing to reorganisation of the observatories or other causes.

The earlier series of hourly differences was kept distinct from the later series until the summarizing stage was reached (§ 9). The sums for the former set were then converted into force, and combined with the sums of the second series, the two being thus incorporated in the rows of the summary sheets.

The hourly differences were entered on the lunar sheets in units of $1'$ (or $10 \cdot 7\ \gamma$), decimals of a minute, or the last figure (unit $1\ \gamma$) of the force, in the printed hourly values, being disregarded.

The results obtained by the final harmonic analysis were reversed in sign, in order to transform them to west force.

§ 12.2. *Horizontal force at Batavia.*—Up to 1899 the printed records give the horizontal force, and afterwards the north force-component. No distinction was made between the two sets of data, owing to the small value of the declination. The force was given to the nearest γ, but the lunar hourly differences were entered on the lunar sheets in units of 10γ.

§ 12.3. *Declination at Zikawei.*—Nearly, but not quite, all the published declination data for Zikawei were used in this investigation ; a few months of the earlier series were not available in the library of the Royal Observatory, Greenwich. The observations after 1908 relate to Lukiapang, the adjacent site to which the original Zikawei observatory was transferred. It was not thought necessary to make any distinction between these two sets of data. The unit in terms of which the Zikawei data were printed was $1'$ after 1899, but from 1877 to 1899 it was $0' \cdot 63$, tenths of the unit being recorded. As in the Batavian and Greenwich reductions, these tenths were ignored in entering the hourly differences on the lunar sheets. The two sets of data expressed in different units were kept separate till the summarizing stage was reached ; the sums from the earlier data were then converted to the unit $1'$, and combined with the later sums. After the final stage, of harmonic analysis, the results were expressed in force units, westerly force being positive.

§ 12.4. *Declination at Pavlovsk.*—The Pavlovsk declination data were expressed in arc to $0' \cdot 1$; the decimals of a minute were ignored in entering the hourly differences on the lunar sheets. A slight complication arose from the practice, which was followed at Pavlovsk for some years, of printing hourly differences from the daily mean, instead of actual hourly values, or hourly differences from the monthly mean. The variation of the daily mean from day to day had thus to be taken into account. This was a relatively easy matter owing to the separation of the lunar daily sequences according to their solar hour of commencement (§ 3) : for the difference between successive daily means was applicable as a correction to the *same* columns for all the rows on any one of the 24 lunar sheets in each section of the data. Consequently it was only necessary to use the *sum* of the differences between the daily means, for all the sequences contributing to a given row of total or group sums on any lunar sheet, as a correction to part of that row of sums.

By an oversight in executing the reduction of the Pavlovsk declination data, the solar diurnal magnetic variation was not abstracted before re-writing the hourly values (*not* differences, in this case) on the lunar sheets. In all the other reductions (§§ 12, 1–3, 5) S was abstracted, as described in § 3.

The method of reduction described in §§ 9, 10 should nevertheless eliminate S from the final sums I–IV, provided that on the summary sheets the 24 rows of sums all refer to equal numbers of days ; but departures from equality in this respect will affect the determinations of L_r more seriously in this case (owing to the relatively large amplitude of S) than when S has initially been removed. On most of the summary sheets this will only increase the accidental error of the resulting values of L_r, because no systematic variations in the numbers of days included in the different rows of sums are likely ; but on the summary sheets on which the lunar days are separated according to the moon's distance, into the groups A, *a*, *p*, P (§ 4), such systematic variations are more likely (owing to the near agreement between the lengths of the synodic and anomalistic months), and did, in fact, appear. The results from these summary sheets are therefore not given in this paper.

During part of the years 1882–3, Göttingen time was used at Pavlovsk instead of local time. The corresponding data were omitted from this investigation.

§ 12.5. *Horizontal and Vertical Force at Pavlovsk.*—The printed records gave the horizontal and vertical components of force to 1 γ, but the last digit of the lunar hourly differences was ignored, so that the unit on the lunar sheets was 10 γ. The solar diurnal variation was abstracted as in other cases, and the complication mentioned in the first paragraph of § 12.4 was met as in the case of the declination.

Use of Greenwich Times of Lunar Transit.

§ 13. In re-ordering the data according to lunar time, the solar hour (in local time) of the initial member of each lunar daily sequence should have been the exact hour nearest to the local time of lunar transit at the given observatory ; but it was more convenient to use, instead of the local solar time of local lunar transit, the Greenwich solar time of Greenwich lunar transit on the same day. The two times differ only slightly, and the Greenwich times can be taken directly from the *Nautical Almanac*. This practice —adopted in nearly all the present cases (though not in the Pavlovsk horizontal force reductions)—necessitates a small additional correction to the phases of L_r ($r = 1, 2, 3, 4$) finally obtained. The correction is of amount $+ 2\ L°/29$ (the same for all components), where L is the longitude of the station, in degrees, from Greenwich, reckoned positive for westerly and negative for easterly stations.*

The Classification of Days according to Lunar Distance.

§ 14. This classification was made as in § 4, precisely the same as for Greenwich.

* *Cf.* 'Phil. Trans.,' A, vol. 214, p. 295, 1914, where, however, the correction is given with the wrong sign. The error was pointed out in a later paper.

The Classification according to Range or Magnetic Activity.

§ 15. This classification was made in substantially the same way as for the Greenwich data (§§ 5, 7, 8). The days were combined into four groups, denoted by Q, O_1, O_2, and X; Q and X represented quiet and disturbed days, at the rate of 5 each per month, so far as the method of classification permitted; groups O_1 and O_2, of less or greater intermediate range, each contained about 10 days per month.

§ 15.1. For the declination at Batavia the groups Q, O_1, O_2, and X respectively contained 431, 1123, 966 and 641 days out of a total of 3161. They corresponded to range $1'$ (Q), $2'$ (O_1), $3'$ (O_2), and greater than $3'$ (X), or, in force units, 11γ, 21γ, 32γ, and more than 32γ. These limiting ranges, in force units, for the quiet and disturbed groups agree fairly well with the corresponding limits for the Greenwich data (§ 8).

§ 15.2. The frequency table for the horizontal force at Batavia, the range being in units of 10γ, was as follows:—

Range	1	2	3	4	5	6	7	>7	All
Number of days ..	100	818	1051	537	264	136	78	165	3149

The activity-groups were therefore chosen as follows:—

Group	Q	O_1	O_2	X
Ranges	1, 2	3	4	5, 6, 7
Number of days	918	1051	537	478

The most highly disturbed days (of range more than 70γ) were not included in group X. It should also be noted that group Q here contains nine days per month.

§ 15.3. Two frequency tables for the ranges of Zikawei declination were formed, for the two sets of data in different units (§ 12.3). In these tables all months and years were taken together. The results were as follows:—

Range		0	1	2	3	4	5	6	7	8	9	10	11	All
Number of days	Unit $0'{\cdot}63 = 6\gamma$	0	8	118	301	302	207	131	51	23	15	4	4	1164
	Unit $1' = 9{\cdot}6\gamma$	6	299	1002	737	252	58	23	10	5	1	2	2	2397

The 3561 lunar daily sequences were classified as follows:—

Group.	Ranges (first set)	Ranges (second set)	Total number of days.
Q	0, 1, 2	0, 1	431
O_1	3	2	1303
O_2	4, 5	3	1246
X	>5	>3	581

§ 15.4. The frequency table for Pavlovsk declination is not comparable with the other frequency tables, because the daily range is enlarged by the inclusion of S; it is therefore not reproduced here. The activity-groups into which the 4172 lunar daily sequences were divided were as follows :—

Group.	Ranges.	Number of days.
Q	<10	423
O_1	10, 11, 12	1491
O_2	13–16	1591
X	>16	667

§ 15.5. The frequency table for Pavlovsk vertical force indicated the following choice of range limits for the activity-groups :—

Group	Q	O_1	O_2	X	All
Ranges (unit 10 γ)	0, 1	2	3	$\geqslant 4$	
Number of days	1232	1603	564	741	4140

§ 15.6. The frequency table for Pavlovsk horizontal force was as follows :—

Range (unit 10 γ) ..	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of days ..	3	214	1312	1094	687	377	206	164	106	46	44	25	23

Range ..	13	14	15	16	17	18	19	20	$\geqslant 21$
Number of days	10	8	3	11	2	7	3	3	30

Eighteen days of range over 250 γ were omitted altogether from the work. The remaining 4360 days were grouped as follows :—

Group	Q	O_1	O_2	X	Total.
Ranges (unit 10 γ) ..	$\leqslant 2$	3	4, 5	>5	
Number of days	1529	1094	1064	673	4360

Description of the Summary Sheets.

§ 16. The summary sheets relative to the data considered in the present part of this paper were as follows : one for each calendar month (combining all years), that is, four monthly summary sheets for each set of data ; four for the lunar distance groups A, *a*, *p*, P, combining all months and years ; four for the activity-groups Q, O_1, O_2, X, also combining all months and years.

Part III.—Description of the Results.

Verification of the Law of Change of Phase.

§ 17. The changes of phase in the components L_r of L were originally discovered empirically,* and subsequently explained theoretically (in the same paper) on the hypothesis that L is a manifestation of electric currents in the atmosphere induced by a lunar semidiurnal tidal atmospheric oscillation, and flowing in an atmospheric layer of which the electrical conductivity depends on the sun's hour-angle.

The reality of these changes of phase in L_r is scarcely open to doubt, but the present

* 'Phil. Trans.,' A, vol. 213, p. 279, 1913.

extensive investigation of the Greenwich declination data affords a good opportunity of indicating to what degree they are apparent in the results, and how far they are blurred by accidental irregularities. The whole amplitude of L is, of course, very small.

For this purpose the complete series of 20,762 rows of lunar hourly differences were combined together into 24 groups according to their solar commencing hour or lunar age, ignoring all other differences, of season, lunar distance, and so on. Each of the 24 groups contained about 850 lunar daily sequences.

A row of totals was formed for each group, and was analysed in order to obtain the amplitude and phase of the first four components of L, according to the formula

$$\Sigma \, C_n \sin (nt + \theta_n)$$

as in § 11, except that here the phases were *not* reduced to the epoch of new moon. The amplitudes were expressed in units of $0 \cdot 01 \, \gamma$ or 10^{-7} c.g.s. The results are given in Table III.

TABLE III.

Solar hours of commencement of lunar day.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
		°		°		°		°
0	69	128	102	61	43	138	5	99
1	57	117	95	67	34	107	19	144
2	73	122	122	66	33	141	17	213
3	23	105	126	69	35	140	24	204
4	40	86	131	64	29	178	26	263
5	19	43	117	72	18	177	22	286
6	72	4	113	41	43	198	11	195
7	65	20	131	57	11	207	8	330
8	47	41	106	66	42	232	16	284
9	67	1	118	56	33	252	15	48
10	45	28	114	71	32	254	12	160
11	55	352	111	55	41	284	17	80
12	44	284	111	51	27	312	18	93
13	53	259	101	55	43	307	12	211
14	82	276	117	64	17	301	18	174
15	56	269	124	56	42	349	11	100
16	88	258	122	67	38	11	13	117
17	31	245	106	59	27	338	18	178
18	26	237	108	46	27	36	6	359
19	29	176	119	46	32	18	21	335
20	57	202	131	63	25	85	20	315
21	64	190	101	51	30	48	16	76
22	65	126	101	40	34	90	23	91
23	27	124	127	70	40	120	15	93

According to the above-mentioned theory the values of C in each column, and also the values of θ_2, should be constant, while θ_1 and θ_3 should successively decrease and increase respectively by 15° from one row to the next, and θ_4 should increase by double this amount. Bearing in mind the smallness of the unit in which the values of C are expressed, the values of C_2 and θ_2 agree satisfactorily with expectation; the extreme range in θ_2 is 31°, and the extreme percentage range of C_2 on either side of the mean is 16 per cent., or, in absolute measure, 0·18 γ. The range in C_1 is larger, both absolutely and proportionately, but even here the greatest departure from the mean is very small, 0·40 γ, while the mean departure from the mean is only 0·17 γ. The ranges in C_3 and C_4 are actually very small, though the mean values of these amplitudes are so small that the proportional variations are by no means negligible.

The degree of regularity of the phase changes in Table III can most easily be perceived by reference to fig. 4, where the phases are represented as varying in a continuous way, so that, for example, 0° may be shown either as 0° or 360° or 720°. The straight lines are drawn to indicate the variations theoretically expected: in order to show that the phase θ_4 changes at double the rate of θ_1 and θ_3, the vertical scale of angle for θ_4 is in the figure taken as half that for θ_1, θ_2, θ_3 (this reduction of the scale of angle for θ_4 is also appropriate in view of the very small amplitude C_4 of L_4, which, of course, makes the significance, in force units, of a given accidental variation in θ_4 much less than that of the same variation in θ_1, θ_2, or θ_3). It is clear from the figure that the phase-angles θ_1, θ_3 and θ_4 do change at the theoretical rates and in the expected directions.

In the theory of these changes of phase which was given in the paper already referred to in this section, it was concluded that all the phases should agree at new moon. The last line of Table IV indicates that this is not the case. The result is not surprising, however, in view of the fact that L must be a distinctly more complex phenomenon (*cf.* Part IV of this paper) than was assumed in the theory referred to.

The Seasonal Changes of the Lunar Diurnal Variation.

§ 18.1. In Table IV the amplitudes and phases of the first four components of the lunar diurnal variation of declination at Greenwich are given for each calendar month, for the three seasons, and for the mean of the year, as derived from the whole material used. The number of lunar days from which each set of values are derived is indicated, and these numbers are so great as to give considerable confidence in the results, at least in cases where the amplitude C is 10 units (0·1 γ) or more. Where the amplitude is less than this, very little reliance can be placed on the corresponding phase.

With Table IV it is of interest to compare Table V, which gives the corresponding results for the *solar* diurnal variation of Greenwich declination, derived from the same 63 years. The unit in Table V is ten times that used in Table IV, namely 0·1 γ instead of 0·01 γ. The harmonic formula used is the same as that of § 11, except that here t refers to *solar* time reckoned from midday (*apparent*, not *mean* midday).

The main component of L in the declination at Greenwich is the second (L_2), whereas in S it is the first (S_1). The amplitude of L_2 is about one-ninth that of S_2, but in the other components (particularly the first) the ratio of L_r to S_r is still smaller.

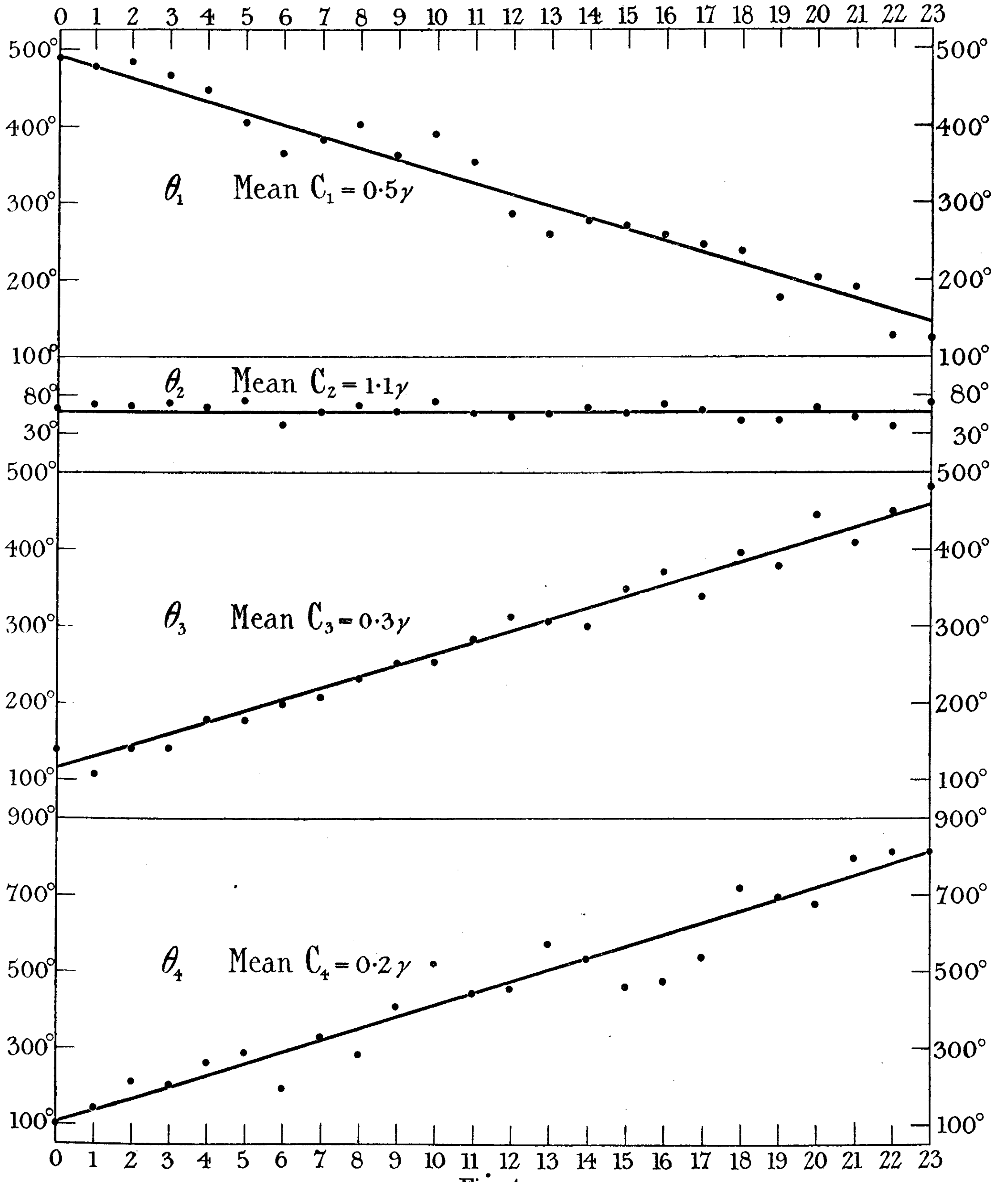


Fig. 4.

The seasonal changes of amplitude are relatively greater for L than for S, and the seasonal changes of phase are also greater. This characteristic, as the writer has remarked elsewhere, is shown likewise by the results for other observatories and elements. Most of the components of L and S are greatest in summer and least in winter, but the fourth component, both of L and S, is greatest at the equinoxes.

TABLE IV.—Lunar Diurnal Variation of Greenwich Declination (*cf.* § 11). Unit $0 \cdot 01\ \gamma$.

	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
			°		°		°		°
January . . .	1735	45	274	77	357	13	315	7	189
February . .	1621	66	219	70	32	15	130	7	79
March . . .	1746	26	97	118	54	36	105	37	88
April	1733	54	109	137	50	54	95	17	114
May	1763	85	102	211	53	65	83	15	110
June	1702	92	115	184	62	52	82	14	217
July	1761	120	117	167	78	38	82	10	29
August . . .	1772	114	135	160	91	64	127	8	61
September . .	1663	131	148	126	109	67	134	16	97
October . .	1774	60	163	95	64	33	160	25	117
November .	1743	6	225	87	33	6	305	12	135
December . .	1749	53	263	82	19	4	10	4	338
Summer . . .	6998	101	119	174	70	52	94	5	100
Equinox . .	6916	62	139	109	68	43	122	23	101
Winter . . .	6848	38	249	77	20	1	—	4	130
Year	20,762	48	139	114	59	31	107	10	105

TABLE V.—Solar Diurnal Variation of Greenwich Declination. (The unit is ten times that of Table IV.)

	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
		°		°		°		°
January	98	80·0	51	23·6	22	78·3	17	57·4
February	116	73·9	67	28·5	32	61·2	19	48·0
March	157	62·0	105	40·0	62	54·4	23	60·0
April	169	50·7	132	37·1	63	46·7	19	64·9
May	172	44·6	124	46·8	45	62·8	10	93·4
June	186	36·1	126	46·0	38	62·4	2	96·4
July	181	39·6	119	48·6	40	59·8	4	23·5
August	174	44·5	128	56·3	53	66·6	7	53·5
September	163	63·3	116	46·8	53	60·7	20	69·8
October	134	67·0	94	26·3	48	43·6	26	52·3
November	104	77·4	61	18·3	28	60·1	18	43·7
December	89	85·0	43	15·1	17	74·1	13	49·0
Summer	178	41·2	124	49·5	44	63·2	5	69·6
Equinox	155	60·8	112	37·6	56	50·9	22	61·4
Winter	102	79·1	55	21·4	24	66·4	16	49·1
Year	140	57·6	95	40·1	41	58·1	14	58·4

§ 18.2. The corresponding results for L at Batavia, Zikawei, and Pavlovsk, so far as they have been obtained in this paper, are given in Table VI, in the same manner as for Greenwich declination in Table IV.

TABLE VI.—L. Unit 10^{-7} c.g.s. ($0 \cdot 01\ \gamma$).

Month.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
				Batavia West declination.					
November . .	809	120	235°	204	255°	148	278°	51	289°
December . .	806	148	254	259	267	186	287	61	306
January . .	782	225	263	372	283	260	302	75	331
February . .	764	157	243	268	259	206	278	71	293
(S) summer .	3161	159	251	270	268	196	288	61	306
				Zikawei West declination.					
May	939	70	67	213	74	122	81	24	119
June	863	123	70	254	79	128	90	21	160
July	881	163	85	277	93	151	100	15	136
August . . .	878	123	87	272	100	138	111	19	156
(N) summer .	3561	118	78	248	88	132	97	18	142
				Pavlovsk West declination.					
May	1058	70	168	69	41	11	195	63	254
June	1027	200	169	87	115	47	210	51	241
July	1060	233	146	130	124	42	186	67	245
August . . .	1027	221	130	185	119	51	151	43	279
(N) summer .	4172	174	149	104	110	31	188	54	253
				Batavia horizontal force.					
November . .	805	95	148	124	187	35	142	38	271
December . .	805	76	196	118	211	56	211	16	229
January . .	807	110	186	133	214	53	206	25	231
February . .	732	105	182	113	195	58	197	8	133
(S) summer .	3149	92	178	120	202	46	195	14	222
				Pavlovsk horizontal force.					
May	1103	68	5	96	3	95	343	8	39
June	1067	60	26	103	2	64	338	14	136
July	1097	98	24	126	17	52	356	29	246
August . . .	1093	101	24	90	6	80	3	6	347
(N) summer .	4360	79	17	103	8	72	350	7	278
				Pavlovsk vertical force.					
May	1041	62	207	26	287	7	10	10	99
June	1013	37	157	18	2	7	336	11	185
July	1041	96	126	21	29	24	174	8	279
August . . .	1045	4	166	10	65	9	281	13	209
(N) summer .	4140	42	157	10	65	3	230	6	194

These results will not be discussed here in detail, but it is of interest to compare them with those formerly obtained by the writer for the same observatories and elements, for a shorter period of years,* and by a different method of analysis. Only the means for the four-monthly period in each case can be compared. The results are as follows: p referring to the present and f to the former† set of results.

TABLE VII.

		C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
			°		°		°		°
Batavia West force or declination	p	159	251	270	268	196	288	61	306
	f	109	233	260	268	135	293	67	310
Zikawei West force or declination	p	118	78	248	88	132	97	18	142
	f	117	81	236	83	117	94	14	130
Pavlovsk West force or declination	p	174	149	104	110	31	188	54	253
	f	131	114	128	88	28	83	2	—
Batavia Horizontal or north force	p	92	178	120	202	46	195	14	222
	f	31	130	141	210	48	208	23	219
Pavlovsk Horizontal or north force	p	79	17	103	8	72	350	7	278
	f	61	20	105	4	62	355	26	291
Pavlovsk Vertical force	p	42	157	10	65	3	230	6	194
	f	36	145	13	203	11	282	1	—

While there are some discrepancies between the above two sets of results, it seems fair to say that, in view of (i) the extreme smallness of the unit, (ii) the difference in the periods of years used (the period formerly dealt with having been only one-third or one-quarter that dealt with in this paper), and (iii) the difference in the methods of analysis, the degree of accordance shown by the Table is distinctly satisfactory.

The Influence of Changes in the Lunar Distance.

§ 19. In a former paper‡ I have given an account of the attempts to determine in what way L changes as the moon's distance varies, and in the same paper I discussed

* This was 1897–1903, except for Batavia, for which the years 1888–1890 replaced the years 1899–1901, during which the observatory was being re-organised.

† 'Phil. Trans.,' A, vol. 218, pp. 113, 114. Note that the *northern* seasons are there referred to.

‡ 'Phil. Trans.,' A, vol. 215, pp. 161–176, 1915.

the same question on the basis of new data. All such discussions hitherto have been concerned only with the second component (L_2) of L, and in the paper quoted I confined my attention to cases where the semi-amplitude C_2 of L_2 for the elements considered was 1 γ or more. In the present paper data are available for a similar discussion concerning the other components L_1, L_3, and L_4, though, of course, unless C in these cases is nearly equal to 1 γ (or 100 of the units to which the tables refer), the proportionate changes caused by variations in the lunar distance will be too small in absolute magnitude to be reliably shown by the data.

In the paper quoted I concluded that C_2 varied with lunar distance nearly (though possibly not quite to the full extent) in inverse proportion to the cube of the distance; that is, C_2 varied nearly as the moon's tide-producing force. Moreover, the phase θ_2 appeared to increase from apogee to perigee by an amount approaching 25° or 30°; when half months centred at apogee and perigee were considered, this change was reduced to about 9°.

These conclusions can now be reviewed in the light of the following new data, obtained as has been described in Parts I and II. In general, only those results relating to cases where C_2 is 0·5 γ or more are reproduced here.

For Greenwich the quiet and disturbed days were not investigated from this standpoint, and the following results refer to days of "ordinary" activity O (§ 9). The first six lines of the table relate to all months together; the letters P, A, *p*, *a*, are explained in § 4, while P*p*, A*a* refer to the half-lunations centred at perigee and apogee respectively.

TABLE. VIII.—Greenwich West declination. (Force units, 0·01 γ.)

Group.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.
			°		°		°
P (perigee)	2252	21	168	136	69	49	104
p	3820	78	137	129	56	40	102
a	3810	46	135	106	53	34	92
A (apogee)	2294	58	133	95	56	44	90
P*p*	6072	56	141	130	61	43	103
A*a*	6104	50	134	102	54	38	91
Summer P*p*	2032	136	117	208	74	68	94
,, A*a*	2037	100	117	167	67	61	85
Equinox P*p*	1792	68	137	129	64	60	110
,, A*a*	1803	69	147	101	61	53	104
Winter P*p*	2248	—	—	81	26	—	—
,, A*a*	2264	—	—	66	11	—	—

The results for Batavia, Zikawei, and Pavlovsk (omitting the declination results for the A, a, p, P groups in the latter case, for the reason mentioned in § 12.4, and the vertical force results, because of their small amplitude) are as follows :—

TABLE IX.

Group.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
				Batavia West declination.					
P	584	191	268°	331	283°	241	301°	67	323°
p	987	176	246	292	269	208	291	73	311
a	1012	152	249	252	261	186	281	54	301
A	578	120	235	218	259	163	276	55	289
				Zikawei West declination.					
P	658	130	85	282	98	155	112	—	—
p	1141	132	85	262	90	132	100	—	—
a	1130	106	72	239	83	131	91	—	—
A	632	104	65	218	78	126	82	—	—
				Batavia horizontal force.					
P	588	50	214	115	214	—	—	—	—
p	978	70	192	142	204	—	—	—	—
a	990	113	170	106	199	—	—	—	—
A	593	155	166	110	192	—	—	—	—
				Pavlovsk horizontal force.					
P	816	85	45	127	4	88	343	—	—
p	1353	99	17	112	5	75	4	—	—
a	1365	57	359	95	14	67	341	—	—
A	826	94	6	80	6	63	346	—	—

In these results all days have been used which were used in obtaining the results of § 18. As a further check, the data for declination at Batavia, Zikawei and Pavlovsk were subdivided in another way, keeping the less disturbed days (of activity-groups Q, O_1) separate from the more disturbed days (of groups O_2, X), but taking together the A and a, or the p and P days, that is, considering half-lunations only. This subdivision yielded the following results :—

TABLE X.

Group.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
				Batavia West declination.					
Quiet Pp . .	712	76	224°	154	262°	115	283°	40	300°
,, Aa . .	842	40	219	116	254	95	277	31	282
Disturbed Pp	859	276	261	432	278	310	299	97	320
,, Aa	748	257	249	380	262	269	280	83	303

TABLE X—*continued.*

Group.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
				Zikawei West declination.					
			°		°		°		°
Quiet P*p* . .	852	—	—	148	88	68	95	—	—
,, A*a* . .	882	—	—	131	80	74	79	—	—
Disturbed P*p*	947	214	87	379	95	206	107	—	—
,, A*a*	880	172	65	333	82	184	91	—	—
				Pavlovsk West declination.					
Quiet P*p* . .	938	174	245	57	338	—	—	69	255
,, A*a* . .	976	173	221	45	290	—	—	58	241
Disturbed P*p*	1097	349	126	233	122	60	156	—	—
,, A*a*	1161	331	122	231	108	53	155	—	—

The most remarkable feature of this table is, of course, the great difference between the amplitudes of the various components of L on disturbed and on quiet days: this will be considered further in § 20; but another noticeable feature is the greater consistency of the changes due to lunar distance, as compared with those shown in Table IX; it would seem that the mixture of days of different activities, as in the groups of data considered in Table IX, appreciably increases the accidental error of the determinations, particularly of those components whose phase changes throughout the lunation, viz., L_1, L_3, L_4. This, of course, is not surprising.

The results for the separate P, *p*, *a*, A groups show, for the second component L_2, a decided tendency for both amplitude and phase to increase from apogee to perigee. The separate results may conveniently be combined by giving to each value of C_2 a weight proportional to the number of days from which it has been derived, the weighted mean for each group, P, *p*, *a*, A then being formed. The departures from the weighted mean of the four groups combined can then be formed. The result is as follows:—

TABLE XI.

Percentage Variation of C_2 with Lunar Distance. Combined result from Tables VIII, IX.

Group.	Number of days.	Percentage deviation from mean.	Phase change.
P	4898	+15	+9·5
p	8279	+ 9	−0·3
a	8307	− 7	−3·7
A	4923	−18	−6·0

The above table also contains, in the last column, the weighted mean of the phase-differences from the mean, the weight attached to the phase-difference in each set being approximately proportional to the contribution of that set to the weighted mean of C_2.

These results confirm those previously obtained; the amplitude-change between the groups A and P appears to be about 33 per cent., which is very close to the ratio of the mean tide-producing forces for these groups, namely, 37 per cent. The phase change, $15^\circ \cdot 5$, is of the same sign as, but rather smaller magnitude than, that found in the earlier paper.

The Greenwich results for the half-lunations (groups P*p*, A*a*) and the results of Table X are also very consistent in indicating these changes. The amplitude and phase changes —the former expressed as percentages of the mean amplitudes—may be summarised as follows:—

TABLE XII.—Variation of C_2 and θ_2 with lunar distance: half-lunations.

	C_2.	θ_2.		C_2.	θ_2.
	Percentage.	°		Percentage.	°
Greenwich: Summer	23	7	Batavia: Quiet	24	8
Equinox	24	3	Disturbed	13	16
Winter	21	15	Zikawei: Quiet	13	8
			Disturbed	13	13
			Pavlovsk: Quiet	23	48
Year	23	7	Disturbed	1	14

The weighted mean of these results gives a percentage change of amplitude but little less than 23 per cent., which is the change in the mean tide-producing force from one half-lunation to the other. The phase-change is, in the mean, about 10°; the large value for Pavlovsk declination on quiet days corresponds only to a small value of C_2.

The results for the other components of L are less consistent among themselves, partly owing to their smaller amplitude. The results of Table X are the most definite in indicating that L_1, L_3 and L_4 are also increased in amplitude and phase at perigee, and not improbably to an extent similar to L_2. Tables VIII and IX do not disagree with Table X, but the results, especially those for C_1, are much more irregular. The mean results from the disturbed day groups of Table X, for which the amplitudes are largest, are as follows:

TABLE XIII.—Variations of L_1, L_3 with lunar distance: half-lunations.
Disturbed-day groups: declination.

	C_1.	θ_1.	C_3.	θ_3.
	Percentage.	°	Percentage.	°
Batavia	7	12	14	19
Zikawei	22	22	11	16
Pavlovsk	5	4	12	1
Mean	11	13	12	12

The Relation with Magnetic Activity.

§ 20. The results given in Table X have already shown that L, in all its components, is increased in amplitude at times when the magnetic activity is above the average. The following tables contain further data illustrating this effect more in detail. Table XIV refers to Greenwich declination, while Table XV relates to the data from which, in other groupings, Table VI is derived. Table XV includes the results of Table X, though the fact that the activity-change in L is so consistently shown by the sub-groups of Table X is further evidence of its reality.

TABLE XIV.—Greenwich West declination (force units, $0 \cdot 01\ \gamma$).

Season.	Activity group.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
				°		°		°		°
Summer .	Q	1627	62	124	106	68	26	106	1	—
	O	4069	118	117	187	71	64	90	6	78
	X	1123	106	116	233	69	50	97	13	154
Equinox	Q	1559	28	144	71	57	37	121	14	103
	O	3595	68	143	114	63	56	107	21	97
	X	1439	44	146	120	79	53	147	35	94
Winter .	Q	1089	8	219	56	10	1	—	3	—
	O	4512	34	244	73	19	6	94	4	—
	X	1013	79	257	98	31	21	333	20	119
Year . .	Q	4275	34	134	75	55	23	115	4	100
	O	12176	53	137	116	58	40	97	9	101
	X	3575	39	152	143	65	29	121	22	109
	XX	736	109	139	164	59	64	233	17	129
	All	20762	48	139	114	59	31	107	10	105

TABLE XV.

Activity group.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
			Batavia West declination.						
			°		°		°		°
Q	431	18	162	52	245	56	279	22	319
O_1	1123	75	218	164	260	124	289	41	306
O_2	966	190	240	317	267	234	292	74	316
X	641	388	255	542	275	368	303	111	338
			Zikawei West declination.						
Q	431	2	—	69	85	26	93	9	227
O_1	1303	54	78	162	84	84	94	3	—
O_2	1246	156	68	308	88	174	105	31	157
X	581	263	73	445	90	229	111	37	146

TABLE XV—*continued*.

Activity group.	Number of days.	c_1.	θ_1.	c_2.	θ_2.	c_3.	θ_3.	c_4.	θ_4.
			Pavlovsk West declination.						
			°		°		°		
Q	423	363	236	204	284	88	281	65	261
O_1	1491	117	217	33	25	24	252	63	264
O_2	1591	275	128	188	122	61	171	48	266
X	667	516	104	341	104	53	138	50	286
			Batavia horizontal force.						
Q	918	50	172	78	213	33	188	1	—
O_1	1051	61	173	120	206	45	183	20	235
O_2	537	138	179	180	203	73	185	26	189
X	478	169	169	163	199	85	212	25	218
			Pavlovsk horizontal force.						
Q	1533	55	3	79	7	52	345	12	277
O_1	1095	111	5	118	8	74	343	8	318
O_2	1063	102	31	96	10	84	347	14	251
X	669	65	42	147	5	97	344	10	112
			Pavlovsk vertical force.						
Q	1231	28	134	3	79	5	262	2	—
O_1	1603	34	115	13	310	10	321	8	216
O_2	565	49	94	21	288	4	279	17	214
X	741	155	198	49	31	33	155	14	116

These tables show with considerable consistency that L varies with the degree of magnetic activity far more than with any other factor, whether season, lunar age or distance, or, as § 21 indicates, sunspot epoch. In all the above cases, except that of Pavlovsk declination, the change is of an extremely simple character, common to all components; the phase is nearly constant (where the corresponding amplitude is sufficiently large) whatever the magnetic activity, but the amplitude steadily increases with the activity. The ratio of increase varies, however, with the locality and the element in question. This is most readily seen by comparing the amplitudes for quiet and disturbed day groups, chosen, as has so far as practicable been done, each to include about one-sixth of all the days. Where the groups are larger or smaller than this, the difference must be taken into account in the comparisons.

For Batavia West force or declination the aforesaid ratio is of the order 10, at Zikawei it diminishes to about 6, and at Greenwich it is reduced to about 2. Proceeding further north still, to Pavlovsk, a change of a somewhat different kind is observable in the declination: the amplitude of L_2 first *decreases* with increasing magnetic activity, and then increases again *with reversed sign*; the reversal shows itself as a change of phase by 180°, from 284° to 104°. Also in the first and third harmonic components of L for the declination at Pavlovsk the amplitude first decreases and then increases again, but the phase change is rather less than 180° ; this may, however, be merely or partly accidental error. Only the fourth harmonic component preserves its phase unchanged, and its amplitude also shows no decided change.

The change in L for the horizontal (north) magnetic force, as shown by the results for Batavia and Pavlovsk, is of smaller magnitude, the ratio of the disturbed and quiet day values of C_1, C_2, C_3 being approximately 2 in each case. The phases also are constant at each station.

For the vertical force the only results available are those for Pavlovsk, where the magnitude of L in this component is too small to give much confidence in the differences shown by the figures. There is little reason to doubt, however, that the vertical force shows a dependence of L on the magnetic activity, similar in character, if not in amount, to that indicated by the other elements.

Owing to the reversal, or great changes, of phase shown by L for Pavlovsk declination, the mean for all days gives quite a misleading indication of the real amplitude of this variation either on quiet or disturbed days. These phase changes are confirmed by the results for the different and further subdivision of the same data which are given in Table X.

The above results, for declination at all four stations, and for Batavia horizontal force, are also confirmed by being shown for other subdivisions of the same data, as given in Tables XVIII, XIX.

The relation with the Sunspot Epoch.

§ 21. The results of the subdivision of the Greenwich declination data according to the mean sunspot number of the year are given in Table XVI.

TABLE XVI.—L. Unit 0·01 γ.

Season.	Sunspot group of years.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
				°		°		°		°
Summer	α (max.)	1256	102	110	188	66	54	95	9	148
	β . . .	1249	115	116	184	68	64	103	14	137
	γ . . .	1567	114	119	167	74	51	85	5	359
	δ . . .	1358	78	120	158	71	39	92	3	90
	ε (min.)	1568	102	128	179	68	53	88	10	53
	All . .	6998	101	119	174	70	52	94	5	100
Equinox	α . . .	1217	82	141	131	76	47	131	44	104
	β . . .	1208	50	160	120	80	34	137	32	88
	γ . . .	1544	64	150	107	58	57	128	15	65
	δ . . .	1356	70	118	116	68	36	131	23	121
	ε . . .	1591	49	133	85	59	49	93	12	127
	All	6916	62	139	109	68	43	122	23	101

TABLE XVI—*continued.*

Season.	Sunspot group of years.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
Winter .	α . . .	1232	53	212°	70	14°	8	286°	8	100°
	β . . .	1208	27	231	96	33	12	119	9	212
	γ . . .	1512	53	261	81	19	8	143	9	93
	δ . . .	1333	14	257	68	16	10	345	4	155
	ε . . .	1563	57	269	73	17	3	—	3	—
	All . .	6848	38	249	77	20	1	—	4	130
Year . .	α . . .	3705	63	140	121	61	30	112	20	110
	β . . .	3665	51	138	128	64	36	115	14	115
	γ . . .	4623	48	146	111	58	36	112	8	64
	δ . . .	4047	49	123	107	60	22	104	10	122
	ε . . .	4722	38	148	105	55	33	91	7	98
	All . .	20762	48	139	114	59	31	107	10	105

From this table it would seem that L for Greenwich declination increases in amplitude from sunspot minimum to sunspot maximum, but by a small amount only—less than 20 per cent. This is less than the changes caused by variations of lunar distance (§ 19), and far less than those produced by changes in the magnetic activity (§ 20).

The behaviour of L in regard to changes in the general solar activity is remarkably different from the corresponding behaviour of the *solar* diurnal variation S. This is indicated by the following table, which bears the same relation to Table XVI that Table V bears to Table IV. The unit is ten times that of Table XVI.

TABLE XVII.—S. Unit 0·1 γ.

Season.	Sunspot group of years.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
Summer . .	α (max.)	224	43°	159	51°	55	61°	4	101°
	β . . .	190	45	115	42	51	50	7	90
	γ	170	42	115	47	36	66	5	184
	δ	159	44	107	51	38	67	5	61
	ε	142	43	105	50	42	68	7	51
	All . .	178	41	124	50	44	63	5	70
Equinox . .	α	223	55	152	33	59	37	10	76
	β . . .	169	59	122	38	60	49	26	71
	γ	155	62	110	37	55	48	22	61
	δ	142	67	99	43	53	56	30	64
	ε	118	62	88	40	45	54	21	59
	All . .	124	61	112	38	56	51	22	61

TABLE XVII—*continued.*

Season	Sunspot group of years.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
Winter . . .	α	143	70°	71	18°	32	60°	21	47°
	β . . .	117	76	62	22	28	63	19	49
	γ	101	82	57	20	23	65	18	43
	δ	90	86	50	22	21	74	14	48
	ε	70	85	41	29	18	78	12	56
	All . .	102	79	55	21	24	66	16	49
Year	α	194	54	125	38	48	51	11	61
	β . . .	155	58	99	36	46	53	17	66
	γ	136	59	93	38	37	57	12	59
	δ	125	62	84	42	37	64	16	60
	ε	105	59	78	43	35	64	13	57
	All . .	140	58	95	40	41	58	14	58

From this table it appears that S changes by 60 per cent. or 70 per cent., at least in the most important component C_1, from sunspot minimum to sunspot maximum. This change is from three to four times as great as that shown by L.

Neither L nor S show any appreciable change of phase with sunspot epoch.

§ 22. In order to investigate further this difference of behaviour between L and S, the series of years for which data had been used, both at Greenwich and at the other three observatories, were divided into two groups only, of greater and of lesser sunspot activity. For Greenwich the first group included the years in the groups α, β and part of those in group γ; the remaining years (δ, ε and part of γ) were in the second group. For the other observatories a similar division of the years was made, using the sunspot numbers of Table I.

In these two sunspot maximum and sunspot minimum groups of years, the data were further divided according to the magnetic activity, in order to see whether, on days of similar (small or great) activity, the magnitude of L varied appreciably with the sunspot epoch.

The results for Greenwich, for which only the quiet (Q) and disturbed (X) groups of days were considered, were as follows: the numbers refer to L_2, the other components being of very small amplitude.

TABLE XVIII—Greenwich Declination.

	Number of days.	C_2.	θ_2.
Quiet days : Sunspot minimum	2900	78	55°
,, ,, maximum	1375	70	56
Disturbed days : Sunspot minimum	894	127	80
,, ,, maximum . . .	2681	150	61

The changes of amplitude and phase here shown are within the limits of accidental error of the determinations ; they show, at least, that the important change, dependent on the magnetic activity, is scarcely affected by the degree of sunspottedness. This is confirmed by the following results for other observatories, for which a more detailed subdivision of the data, according to magnetic activity, has been made.

TABLE XIX.—L. Unit 0·01 γ.

Activity group.	Sunspot group.	Number of days.	c_1.	θ_1.	c_2.	θ_2.	c_3.	θ_3.
			Batavia West declination.					
				°		°		°
Q	Minimum	242	18	145	40	246	48	279
	Maximum	189	19	185	70	244	69	279
O_1	Minimum	554	68	222	163	259	131	286
	Maximum	569	82	216	166	260	118	292
O_2	Minimum	416	185	241	317	269	251	292
	Maximum	550	194	240	316	265	219	293
X	Minimum	177	412	254	521	280	351	311
	Maximum	464	379	255	551	275	376	300
			Zikawei West declination.					
Q	Minimum	260	7	—	73	89	16	89
	Maximum	171	12	57	65	78	39	95
O_1	Minimum	625	77	79	169	86	83	96
	Maximum	678	34	73	153	82	85	91
O_2	Minimum	485	171	66	334	87	191	106
	Maximum	761	147	69	289	90	162	105
X	Minimum	148	344	77	464	98	277	124
	Maximum	433	235	72	436	87	211	105
			Pavlovsk West declination.					
Q	Minimum	316	317	235	181	288	90	290
	Maximum	107	502	239	279	276	96	256
O_1	Minimum	730	87	137	86	83	21	186
	Maximum	761	231	238	72	311	42	277
O_2	Minimum	360	475	116	229	157	95	153
	Maximum	1231	220	135	191	111	53	180
X	Minimum	98	661	106	393	98	142	152
	Maximum	569	491	103	331	106	38	128

TABLE XIX (*continued*).

Activity group.	Sunspot group.	Number of days.	C_1.	θ_1.	C_2.	θ_2.	C_3.	θ_3.	C_4.	θ_4.
				Batavia horizontal force.						
				°		°		°		°
Q	Minimum . .	495	44	135	80	216	23	175	15	209
	Maximum . .	423	74	196	77	211	45	196	3	228
O_1	Minimum . .	432	55	150	123	203	54	197	25	255
	Maximum . .	619	68	186	118	208	42	169	18	214
O_2	Minimum . .	185	141	203	191	210	83	192	21	255
	Maximum . .	352	145	167	174	198	68	179	36	172
X	Minimum . .	125	187	151	216	210	104	201	40	191
	Maximum . .	353	167	176	147	194	78	214	21	235
All	Minimum . .	1237	69	161	125	209	50	193	20	231
	Maximum . .	1747	103	179	124	202	52	190	17	198

The results of Table XIX may be summarised conveniently by adding together corresponding values of C_1, C_2, C_3 for Batavia, declination and horizontal force, and Zikawei declination (Pavlovsk declination being omitted because of the reversal of phase). The results obtained are as follows :

TABLE XX.

Group.	Number of days.	C_1 (sum.).	C_2 (sum.).	C_3 (sum.).
Q : minimum . .	997	69	193	87
maximum . .	783	105	212	153
O_1 : minimum . .	1611	200	455	268
maximum . .	1866	184	437	245
O_2 : minimum . .	1086	497	842	525
maximum . .	1663	486	779	449
X : minimum . .	450	943	1201	732
maximum . .	1250	781	1134	665

From this table it would seem that in corresponding activity-groups the magnitude of L at these observatories and for these elements is *less* at sunspot maximum than at sunspot minimum, except on quiet days, when, possibly, the reverse is the case. The table shows this with considerable regularity for the different components, though the result can hardly be regarded as definitely established as yet. On account of its remarkable character and important theoretical bearing it is desirable that further efforts be made to confirm or disprove this result. It is not confirmed, but hardly disproved, by the Greenwich results in Table XVIII. The Pavlovsk declination results, which are not included in Table XX, on the whole confirm this table.

I am inclined to credit the result for the following reason. It may readily be verified from the preceding data, that L should show a greater increase, from sunspot minimum to sunspot maximum, than it actually does show, if its amplitude for days of any given range is *independent* of the sunspot epoch. This is merely because of the increase in L with magnetic activity, and the greater frequency and intensity of magnetic activity in years of great sunspottedness. Hence it would seem to follow that there must be a countervailing *decrease* of L, for days of given magnetic activity, at sunspot maximum as compared with sunspot minimum. This decrease must be all the greater, on disturbed days, in so far as the preceding tables are correct in showing an increase of L, for a given daily range, at sunspot maximum, on quiet days.

It may be concluded, then, that L is only slightly affected by sunspot epoch, for days of a given magnetic range; the effect appears to be a slight increase on quiet days, and a slight decrease on more disturbed days, at sunspot maximum as compared with sunspot minimum. It seems absolutely certain that, for a given magnetic activity, L is not increased to anything like the same extent as S (that is, about two-fold—*cf.* Table XVII) from sunspot minimum to sunspot maximum. If this occurred, the mean value of L from all days would be increased more than twofold at sunspot maximum, owing to the then greater frequency of magnetically active days.

PART IV.—SUMMARY AND THEORETICAL DISCUSSION OF THE MAIN FEATURES OF L.

The main features of L.

§ 23. The main features of L, the lunar-diurnal variation of the earth's magnetic field, may be summarised as follows:

(i) At any observatory and for any magnetic element, L, in the mean of a number of whole lunations, is a purely semidiurnal phenomenon. At each stage in the lunation other harmonic components are present, whose amplitudes are usually less than that of the semidiurnal component, while their phases vary so that the average value of these components in the course of a lunation is zero. These components are such as to magnify the intensity of the lunar-diurnal magnetic variations during the hours of sunlight; their phase-variations are due to the regular change of the lunar time of daylight throughout the month.

(ii) It is known, from spherical harmonic analysis, that the magnetic field of L is mainly produced above the earth, though there is a part of internal origin, the character and magnitude of which are such as to suggest that the internal part is a secondary, induced, effect of the primary external varying field. The induction occurs in the oceans and the conducting core of the earth.

(iii) The intensity of the field varies with lunar distance nearly or exactly in the same ratio as the moon's tide-producing force. The phase also changes slightly, being advanced by about 15° (for the semidiurnal component) as the moon passes from apogee to perigee.

(iv) In the middle belt of the earth, the intensity of L increases with increasing magnetic disturbance; but at Pavlovsk the variation in the west component of force decreases, changes sign, and afterwards increases as the magnetic activity increases. The changes of intensity vary with the locality and the magnetic element; for west force at Batavia L_2 on disturbed days has tenfold its value on quiet days.

(v) The mean intensity of the field of L varies but little throughout the sunspot cycle, though there is perhaps a slight increase at sunspot maximum owing to the greater frequency of disturbed days. On equally disturbed days, the intensity of the L field is perhaps slightly *less* at sunspot maximum than at sunspot minimum, the reverse being the case on quiet days.

The tidal origin of L.

§ 24. The features (i), (ii), (iii), together with many detailed characteristics of the distribution of L not here mentioned, receive their simplest explanation in the following way. The magnetic field of L is a consequence of the tide produced in the atmosphere by the moon, and therefore it varies from apogee to perigee (as this tide itself would be expected, and has been shown, to do) in the ratio of the inverse cube of the moon's distance (§ 23, iii). The convective motion of air associated with this tide, in conjunction with the earth's permanent magnetic field, gives rise to electromotive forces throughout the atmosphere. Near the ground the electrical resistance of the air is too great for any appreciable currents to result from these electromotive forces, but in certain higher regions the conductivity rises to a sufficiently high value for currents to flow and to manifest themselves by their magnetic field, which is observed as the lunar diurnal magnetic variation. The external situation of the primary part of the field of L (§ 23, ii) is thus explained.

The theory as thus outlined is similar to that developed by BALFOUR STEWART and SCHUSTER to explain the solar diurnal magnetic variation S. It is probable that the convective motion responsible for S is semidiurnal also, being associated with the well-known semidiurnal variation of the barometer.

Both L and S are more intense during the hours of daylight than during the dark hours, a fact which is readily explained as due to the conductivity of the atmosphere, at the level where the currents flow, being the greater when the air is exposed to some ionising ultra-violet component of the solar radiation.

The marked differences between L and S.

§ 25. A discussion* of L and S along the lines indicated in § 24 showed that the theory was capable of accounting for a great many features of the two phenomena; nevertheless, some serious difficulties (*loc. cit.* § 26) remained. At that time it was not known

* 'Phil. Trans.,' A, vol. 218, p. 1, 1917.

that L and S differ so fundamentally as they have here been shown to do ; these differences indicate that the two phenomena are produced in ways not quite so similar as was assumed in that discussion. The differences and difficulties must receive explanation before the theory can be regarded as complete.

The differences referred to relate to the changes in L and S associated with variations of magnetic activity and sunspot epoch (§ 23, iv, v). Whereas L varies greatly from day to day with the magnetic activity, at the same sunspot epoch, S varies very little, at least over the middle belt of the earth. The variation of S consists of the addition, to the quiet-day solar diurnal variation, of another type of solar-diurnally varying field ; this additional part is ordinarily small over the middle belt of the earth. In the polar regions, on the contrary, this part of S is large, while the former part is relatively insignificant. It is convenient to denote the two parts of S by the symbols S_q and S_d, corresponding to the quiet-day portion, and to the part which depends on magnetic disturbance. Like L, S_d varies considerably in correspondence with the magnetic activity.

But whereas S_q, which in the middle belt of the earth forms the main part of S, is independent of the magnetic activity, it varies regularly with the annual mean sunspottedness or solar epoch, being about twice as large at sunspot maximum as at sunspot minimum. On the other hand, L increases but slightly at sunspot maximum, and this merely because of the increased frequency and degree of magnetic activity or disturbance at this epoch.

§ 26. This analysis of S into two parts S_q and S_d was made by the writer in an earlier paper, and it was inferred that the component of solar radiation which ionises the world-wide conducting layer of the atmosphere wherein S_q is produced must increase greatly in intensity from sunspot minimum to sunspot maximum. It must come from the solar surface as a whole, since it does not show the intermittency, nor the relationship with the solar rotation period, that are manifested by auroræ, magnetic storms, and other phenomena which depend on local disturbances on the sun's surface.

The part of S to which the theory mentioned in § 24 is applicable is S_q. The remaining part, S_d, is most intense and variable in the polar regions, where it would seem to be produced. It represents the part of the field, associated with magnetic disturbance, which is unsymmetrical with respect to the earth's axis—what I have termed the local-time as distinct for the storm-time (or universal-time) part of the magnetic disturbance. It is well-known that there is a close connection between auroræ and magnetic disturbance. Auroræ give visible evidence of ionisation and, consequently, conductivity of the air at high levels (90 km. and more) along the auroral zones and over the polar caps which they delimit. This conductivity must vary greatly from day to day, and these changes will be reflected in, and indicated by, the magnetic disturbance or activity.

§ 27. The variation of L with the magnetic activity receives a simple explanation if L is supposed to be produced mainly in the polar regions. The lunar tidal circulation must exist at all heights in the atmosphere, and the tidal velocity of the air is probably nearly independent of height. It must induce electric currents both in the polar regions and in the S_q layer, but the relative conductivity of the auroral regions must be so much the greater that the currents directly induced in the S_q layer are comparatively insignificant. In this case L will vary in intensity in close correspondence with the variations of magnetic and auroral activity, as it appears to do.

This hypothesis is not without its difficulties, one of which is to explain why the solar semidiurnal convective circulation of the atmosphere does not induce in the auroral zones a correspondingly large part of S. Near the ground the solar semidiurnal atmospheric circulation is about fifteen times as rapid as the lunar one, but the above theory of L requires the supposition that at auroral heights the lunar tidal circulation is the greater ; so far as gravitational tidal effects are unmodified by resonance, this supposition is tenable, because the moon's tidal force is more than twice as great as the sun's. Near the ground the solar semidiurnal circulation is partly due to the sun's thermal influence, and also this oscillatory motion of the air seems to be greatly magnified by resonance. But it is not unlikely that the electromagnetic damping influence to which the oscillation is subjected in the S_q layer makes the motion in the higher regions beyond nearly independent of thermal and resonance effects ; a mathematical examination of this question is now in progress. If the result supports the suggestion here made, the above theory of L should enable an estimate to be made of the conductivity of the auroral zones.