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The Magnetic Storm of December 16–17, 1917, *as Recorded at Kew and Eskdalemuir Observatories.*

By C. Chree, Sc.D., LL.D., F.R.S., Superintendent Kew Observatory.

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§ 1. In a recent paper,* I described a magnetic storm occurring on August 22, 1916, in which the disturbances recorded at Kew and Eskdalemuir Observatories were closely similar in type. The magnetic storm on December 16–17, 1917, affords in many ways a remarkable contrast, the changes recorded at the two observatories differing notably in type. The disturbance was the largest of the year. In the Kew declination (D) and horizontal force (H) curves, a prominent feature was a succession of oscillations with periods of about 20 minutes, and it was the hope of identifying these with corresponding oscillations at Eskdalemuir that suggested the present investigation. Dr. Crichton Mitchell, the Superintendent of Eskdalemuir Observatory, kindly sent the original curves, so the comparison was made under favourable conditions.

One of the obstacles to the identification of corresponding movements at the two stations is that, while H and D are recorded at Kew, the north (N) and west (W) components are recorded at Eskdalemuir. On instituting a minute comparison between the Kew H and D curves, it was found that, while oscillations with periods not far from 20 minutes were prominent in both, the times of the turning points by no means always agreed. A turning point in the H curve, for instance, might be represented by a short arrest or slight temporary reversal of movement in the D curve, the original direction of the D movement being almost immediately resumed. Again, towards the end of an oscillation, the trace might be nearly level for some minutes, or there might be a small short period oscillation, so that more than one choice was possible when assigning a time for the end of one major oscillation or the beginning of the next. Thus, when one had to compare Kew H with Eskdalemuir N curves, and Kew D with Eskdalemuir W curves, the fact that corresponding movements were difficult to identify is not surprising. Both observatories record vertical force (V), but the Kew V curves are so much disturbed by artificial electrical currents that they cannot be relied on for minor details. There were also difficulties special to the particular storm. Some of the N and W movements at Eskdalemuir were so rapid that the photographic trace was in places almost invisible. While the time marks

* 'Roy. Soc. Proc.,' A, vol. 93, p. 177.

occur on the base lines as well as on the curve lines, there is sensible parallax, and when the marks on the curve lines become invisible, the time cannot be fixed with quite the usual accuracy.

§ 2. The disturbance was preceded by a fairly quiet time, so its commencement may be accepted without hesitation as occurring between 8 h. and 9 h. on December 16. But the subsidence was as usual more gradual. Measurements were made of the curves from 8 h. on the 16th to 8 h. on the 17th. They were taken at four-minute intervals, because four minutes answers very nearly to 1 mm. in the base line. The curves were not smoothed, and the arithmetic mean of the measurements made throughout each hour G.M.T. was accepted as the hourly mean. Hourly values were thus obtained from 8·5 h. to 23·5 h. ($11\frac{1}{2}$ P.M.) on December 16, and from 0·5 h. to 7·5 h. on December 17. The arithmetic mean of these 24 hourly values was accepted as the mean for the "day" which commenced at 8 h. on December 16.

Table I.—Mean Hourly Values (Unit $1\gamma \equiv 1 \times 10^{-5}$ C.G.S.).

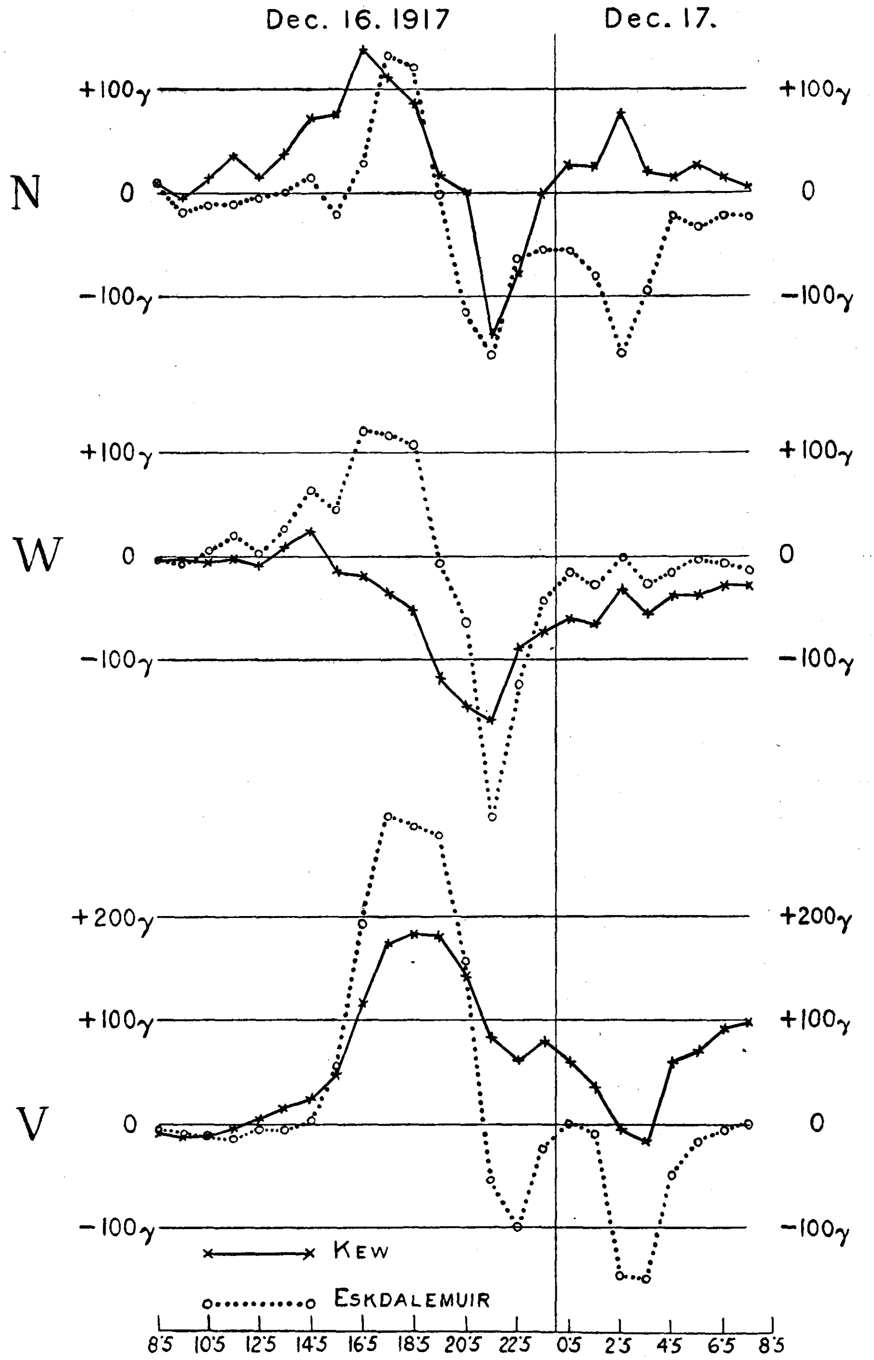
Time, G.M.T.		Kew.			Eskdalemuir.		
		H.	D.	V.	N.	W.	V.
		γ	γ	γ	γ	γ	γ
Dec. 16	8.5 h.	+ 9	− 8	− 8	+ 9	− 3	− 6
	9.5	− 6	− 2	− 13	− 18	− 7	− 10
	10.5	+ 10	− 8	− 13	− 12	+ 4	− 13
	11.5	+ 34	− 10	− 5	− 11	+ 20	− 14
	12.5	+ 12	− 13	+ 6	− 4	+ 2	− 5
	13.5	+ 38	− 2	+ 14	+ 1	+ 27	− 5
	14.5	+ 75	**+ 6**	+ 22	+ 15	+ 64	+ 2
	15.5	+ 70	− 34	+ 48	− 22	+ 46	+ 55
	16.5	**+131**	− 54	+114	+ 28	**+121**	+194
	17.5	+100	− 63	+174	**+132**	+118	**+297**
	18.5	+ 71	− 73	**+184**	+122	+108	+288
	19.5	− 15	−118	+182	− 2	− 6	+279
	20.5	− 36	**−139**	+144	−117	− 65	+158
	21.5	**−175**	−117	+ 82	**−156**	**−252**	− 54
	22.5	−100	− 65	+ 59	− 63	−123	− 99
	23.5	− 21	− 70	+ 80	− 55	− 43	− 24
Dec. 17	0.5	+ 11	− 65	+ 61	− 55	− 14	+ 1
	1·5	+ 6	− 69	+ 35	− 79	− 27	− 8
	2.5	+ 68	− 51	− 8	−153	0	−147
	3.5	+ 4	− 59	**− 19**	− 95	− 27	**−149**
	4.5	+ 4	− 40	+ 59	− 21	− 15	− 47
	5.5	+ 18	− 43	+ 69	− 33	− 3	− 18
	6.5	+ 8	− 31	+ 91	− 21	− 8	− 6
	7.5	− 1	− 28	+ 98	− 22	− 13	+ 1

The Kew D trace records angular changes, but at present a rise of one minute in westerly declination is equivalent to a change of 5·36 γ in force directed 15° south of true west. The Eskdalemuir V trace unfortunately went off the sheet, in the direction of increasing force, several times between 16 h. and 19 h. on the 16th, there being no record for some 55 minutes in all, and the longest continuous absence of trace being about 20 minutes. When the curve was off the sheet, the value answering to the margin of the sheet was accepted. Judging by the trend of the curve, the margin was not much exceeded in most cases. But between 17 h. and 18 h., when the trace was off the sheet for 20 minutes at a time, the margin may have been considerably exceeded. An error x in one hourly mean implies an error $x/24$ in the mean for the day.

The existence of this uncertainty in the mean value of V at Eskdalemuir was one of the reasons which led to the adoption as standard values of quiet day means derived from the previous 24 hours. Table I shows the departure of the hourly means from these quiet day means, the unit being for all the elements $1\gamma \equiv 1 \times 10^{-5}$ C.G.S. magnetic unit. Maximum and minimum values are in heavy type. Every hourly value of D at Kew, except that for 14·5 h., falls short of the quiet day mean, *i.e.*, represents a position of the declination needle to the east of the normal. The existence of this phenomenon at Kew, while there is no corresponding abnormality in W at Eskdalemuir, is one of the most remarkable differences between the two stations. Another remarkable feature is that while N at Eskdalemuir shows the depression customary in that element and in H towards the end of a magnetic storm, there is no corresponding sensible depression in H at Kew. V was much enhanced at both places in the afternoon of the 16th, a customary phenomenon of magnetic storms. This elevation of V was followed by a rapid fall before midnight, also a customary feature; but subsequent to this there was another considerable rise, and a second rapid fall, more especially at Eskdalemuir.

§ 3. Hourly mean values of N and W at Kew were calculated from the hourly means of H and D, and the departures of these from the quiet day means are shown in the accompanying diagram, with the corresponding departures in N and W at Eskdalemuir and in V. The scale of ordinates is the same for the six curves.

Up to 22 h. on December 16 there is a general resemblance between the changes in N at the two stations; but later, especially from 1 h. to 3 h. on the 17th, the changes are more nearly diametrically opposite. The N ranges for the whole 24 hours, as given by the hourly means, were 288 γ at Eskdalemuir as compared with 279 γ at Kew. This gives a very inadequate idea of the excess in the activity at Eskdalemuir. A better idea is given by the values of the absolute range, *i.e.*, the excess of the largest over the least value



recorded at any instant during the 24 hours. These absolute ranges were 620 γ at Eskdalemuir in N, and 408 γ at Kew in H.

The resemblance between the W diagrams for Kew and Eskdalemuir is not close, but it is not less close than usual between 1 h. and 3 h. on the 17th, when the N diagrams from the two stations were opposed. At Eskdalemuir W was markedly enhanced from 13 h. to 19 h. on the 16th, whereas at Kew W was markedly depressed after 15 h. The ranges from the hourly means were 373 γ at Eskdalemuir and 183 γ at Kew. The difference between the absolute W range at Eskdalemuir and the absolute D range at Kew was even greater, the former being 588 γ, the latter only 189 γ.

The ranges from the V hourly means were 446 γ for Eskdalemuir and 197 γ for Kew, the corresponding absolute ranges being respectively 552 γ and 259 γ. As already explained, the values for Eskdalemuir are necessarily under-estimates.

§ 4. Table II gives the results of measurements of the oscillations of longer period already referred to. The different curves were considered and measured quite independently. A minimum reading was taken as the commencement of each oscillation. The second minimum for one oscillation served in many cases as the first minimum for the next. In other cases an interval was left, answering to a portion of the curve which was level or showed one or more short period oscillations, the inclusion of which in either of the two adjacent longer period oscillations would have been arbitrary. If we started with the assurance that we had to do with a continuous train of oscillations, confused by the superposition of waves of shorter period, we should naturally assign the doubtful piece of curve to one or other of the adjacent longer period oscillations. If there had been a very pronounced resemblance between Kew and Eskdalemuir curves, and the hypothesis of a small differential time error had markedly improved the coincidence of turning points, it might have been fairly justified. But, as matters stood, it seemed best to leave the figures exactly as the curve measurements made them.

The result is that 14 longer period oscillations were recognised in both the H and D curves at Kew, the first 13 of which proceeded with little or no interruption, whilst the 14th followed after an hour's interval. The 13th differed somewhat in type from the others. It possibly represented two oscillations rather than one. The mean duration for the first 12 oscillations, accepting the figures in Table II, is in H 20·4 minutes, and in D 21·7 minutes. If we assume that there was a continuous train of oscillations, and accept the times given for the beginning of the 1st and the end of the 12th oscillation, the mean durations become 21·8 minutes for H and 22·5 minutes for D.

At Eskdalemuir, between 15 h. and 20½ h., on the 16th, 16 oscillations of longer period were recognised in N and 13 in W. Some, however, were of

Table II.—Oscillations of Longer Period.

No.	Kew H. 1st min.	Max.	2nd min.	Interval.	Rise.	Fall.	Kew D. 1st min.	Max.	2nd min.	Interval.	Rise.	Fall.
	h. m.	h. m.	h. m.	min.	γ.	γ.	h. m.	h. m.	h. m.	min.	γ.	γ.
1	15 22	15 28	15 36	14	22	14	15 12	15 28	15 36	24	19	42
2	15 36	15 50	15 56	20	50	21	15 36	15 39	15 56	20	19	31
3	16 0	16 13	16 24	24	72	54	15 56	16 5	16 18	22	26	33
4	16 24	16 40	16 44	20	50	45	16 22	16 28	16 44	22	26	32
5	16 48	16 59	17 7	19	59	94	16 44	16 52	17 3	19	28	31
6	17 7	17 17	17 30	23	81	100	17 3	17 17	17 23	20	21	34
7	17 34	17 42	17 48	14	60	24	17 23	17 40	17 46	23	56	22
8	17 48	17 55	18 6	18	27	28	17 46	17 51	18 4	18	17	13
9	18 8	18 12	18 26	18	21	83	18 4	18 6	18 21	17	5	46
10	18 29	18 39	18 52	23	77	67	18 21	18 33	18 50	29	48	43
11	18 52	18 58	19 22	30	27	92	18 50	18 58	19 15	25	5	37
12	19 22	19 28	19 44	22	32	77	19 20	19 26	19 42	22	13	37
13	19 44	20 9	20 25	41	103	77	19 42	19 52	20 13	31	33	34
14	21 13	21 18	21 29	16	83	68	21 10	21 18	21 30	20	65	61

No.	Eskdalemuir N. 1st min.	Max.	2nd min.	Interval.	Rise.	Fall.	Eskdalemuir W. 1st min.	Max.	2nd min.	Interval.	Rise.	Fall.
	h. m.	h. m.	h. m.	min.	γ.	γ.	h. m.	h. m.	2nd min.	min.	γ.	γ.
1	15 11	15 26	15 32	21	64	86	15 13	15 31	15 36	23	53	38
2	15 32	15 40	15 55	23	64	90	15 36	15 51	15 58	22	68	80
3	15 55	16 4	16 7	12	89	25	15 58	16 6	16 11	13	134	37
4	16 7	16 11	16 17	10	61	39	16 11	16 18	16 23	12	80	121
5	16 20	16 24	16 28	8	35	34	16 23	16 41	16 47	24	70	78
6	16 28	16 37	16 44	16	49	34	16 47	17 1	17 7	20	150	192
7	16 44	16 58	17 1	17	143	35	17 7	17 17	17 34	27	203	192
8	17 1	17 4	17 8	7	149	153	17 34	17 42	17 48	14	100	43
9	17 8	17 11	17 14	6	37	45	17 48	17 54	18 8	20	52	48
10	17 19	17 25	17 35	16	184	197	18 8	18 11	18 24	16	22	60
11	17 42	17 50	17 57	15	79	65	18 24	18 35	18 40	16	89	31
12	17 57	18 4	18 14	17	21	105	19 18	19 34	19 46	28	78	145
13	18 18	18 34	18 39	21	250	102	19 46	20 8	20 25	39	108	102
14	18 39	18 45	19 15	36	67	253						
15	19 15	19 34	19 49	34	107	143						
16	19 49	19 53	20 5	16	16	74						

decidedly shorter period than the others, and may have answered to portions only of the oscillations recognised at Kew, large short-period oscillations occurring sometimes at Eskdalemuir while the Kew curve showed almost stationary conditions.

The Eskdalemuir W oscillations could be brought into fair parallelism with the H and D oscillations at Kew by assuming that Nos. 3 and 4 at Eskdalemuir together answered to No. 3 at Kew, whilst Nos. 5–11 at Eskdalemuir corresponded respectively to Nos. 4–10 at Kew, Nos. 12 and 13 at Eskdalemuir corresponding to the oscillations similarly numbered at Kew. We could also bring the earlier N oscillations at Eskdalemuir into fair parallelism with the others by regarding the pairs Nos. 3 and 4, 5 and 6, 7 and 8, and 9 and 10 as each but a single oscillation, answering respectively to Nos. 3, 4, 5, and 6 at Kew.

The Eskdalemuir curves contained some exceedingly large rapid oscillations, practically unrepresented at Kew. These must presumably have arisen from some source of disturbance which was either purely local, or was situated very much nearer to Eskdalemuir than to Kew.

As regards the size of the oscillations recorded in Table II, the mean values of the rise and fall at Kew were respectively 55γ and 60γ for H and 27γ and 35γ for D. The H movements were thus, on the average, decidedly the larger. If we accept the arithmetic mean of the rise and fall as a measure of the amplitude, we find that the first six oscillations in D were very similar in amplitude, while the corresponding H oscillations differed widely. There was, in short, little, if any, parallelism between the amplitudes of the oscillations in the two elements. If we accept the problematical scheme of correspondence between Kew and Eskdalemuir oscillations given above, we find that the Eskdalemuir ranges were almost invariably the larger, but the numerical results, from a comparison of Kew and Eskdalemuir curves, are too uncertain to be worth recording.

§ 5. During many of the rises and falls recorded in Table II, the rate of change of the element varied greatly. Rates of change derived from Table II would thus give an inadequate idea of the rapidity of the more rapid changes. An independent set of measurements was accordingly made, the results of which appear in Tables III–VII. They give the central time of the movement, *i.e.* the arithmetic mean of the times of beginning and ending, the number of minutes the movement lasted, and the estimated rapidity of the change of force in γ's per minute. In most cases, where the rises and falls recorded in Table II showed a sudden marked acceleration or retardation, measurements were made on shorter portions of curve having a nearly uniform slope. In a few cases, however, mean rates are given for longer periods of time, when there was a conspicuous movement in one direction, interrupted by only trifling arrests or reversals. No measurements were attempted of the shorter period movements, mostly small, occupying one or two minutes or less, unless they were quite exceptionally large. The slope of the curve then affords a ready check on the result derived from the time measurements.

Table III.—Rapid Changes in H at Kew.

Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.
h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.
15 8.5	3	−32	11	17 38·0	8	+60	7·5	20 1·0	10	+ 57	5·5
25·0	6	+22	3·5	45·0	6	−24	4·0	11·0	4	− 37	9·0
32·0	8	−14	1·8	51·5	7	+27	4·0	22·5	5	− 41	8·0
43·0	14	+50	3·5	18 0·5	11	−28	2·5	21 9·0	8	−215	27
53·0	6	−21	3·5	9·5	3	+18	6·0	15·5	5	+ 83	17
16 2·5	5	+49	10	19·0	14	−83	6·0	21·0	8	− 27	4·5
12·0	2	+23	11	34·0	10	+77	7·5	27·0	4	− 46	11
21·5	5	−48	9·5	44·0	10	−65	6·5	34·0	2	+ 25	12
27·5	7	+35	5·0	54·0	4	+21	5·0	54·0	10	+ 46	4·5
43·0	2	−35	17	19 4·5	5	−32	6·5	22 32·5	39	+ 89	2·3
53·5	11	+59	5·5	15·5	13	−63	5·0	0 59·0	2	+ 28	14
17 3·0	8	−94	12	24·0	4	+32	8·0	1 3·0	6	− 25	4·0
12·0	10	+81	8·0	38·0	12	−77	6·5	2 14·0	20	+ 99	5·0
18·5	3	−44	15	48·5	9	+53	6·0	55·5	29	−111	4·0
26·5	7	−55	8·0								

Table IV.—Rapid Changes in D at Kew.

Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.
h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.
15 8·5	3	−23	7·5	17 8·0	10	+20	2·0	19 47·0	10	+33	3·5
20·0	16	+19	1·2	20·0	6	−34	5·5	20 6·5	13	−32	2·5
31·0	6	−40	6·5	35·0	10	+48	5·0	21 6·0	8	−35	4·5
37·5	3	+19	6·5	43·0	6	−22	3·5	16·0	4	+54	13
50·0	12	−28	2·3	48·5	5	+17	3·5	21·0	6	−50	8·5
16 0·5	9	+26	3·0	18 15·5	11	−42	4·0	32·5	5	+44	9·0
15·0	6	−30	5·0	27·0	12	+48	4·0	49·0	28	+76	2·5
25·0	6	+26	4·5	38·0	10	−37	3·5	0 59·0	2	+18	9·0
31·5	7	−26	3·5	19 0·5	5	−14	3·0	1 3·0	6	−21	4·0
43·0	2	−14	7·0	11·5	7	−22	3·0	2 41·0	12	+40	3·5
48·0	8	+28	3·5	23·0	6	+13	2·2	58·5	23	−44	1·9
17 0·0	6	−25	4·0	34·0	16	−37	2·3				

Table V.—Rapid Changes in N at Eskdalemuir.

Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.
h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.
15 9·0	4	− 51	13	17 41·0	2	− 30	15	21 19·5	15	−282	19
13·0	4	+ 26	6·5	47·5	5	+ 64	13	31·0	8	+193	24
20·5	11	+ 38	3·5	52·0	4	− 64	16	43·5	7	+ 92	13
29·5	5	− 86	17	18 8·5	5	− 85	17	51·0	6	+ 53	9
36·0	8	+ 64	8·0	26·0	10	+217	22	22 0·0	2	− 28	14
49·5	11	− 89	8·0	33·5	1	+ 32	32	8·5	5	− 30	6
58·5	7	+ 87	12	36·5	5	−102	20	12·0	2	+ 25	12
16 5·5	3	− 25	8·5	39·5	1	+ 29	29	24·0	8	− 45	5·5
9·0	4	+ 61	15	43·0	4	+ 46	11	51·5	7	− 33	4·5
14·0	6	− 39	6·5	47·5	5	− 38	7·5	56·5	3	+ 24	8·0
22·0	4	+ 35	8·5	19 4·0	22	−217	10·0	23 12·0	2	+ 28	14
25·0	2	− 34	17	18·5	7	+ 64	9·0	0 0·0	4	− 29	7
35·5	3	+ 40	13	29·5	3	+ 35	12	59·0	2	+ 37	18
38·0	2	− 22	11	37·0	6	− 79	13	1 2·0	4	− 45	11
43·0	2	− 30	15	45·5	7	− 84	12	51·5	3	− 31	10
49·0	10	+109	11	51·0	4	+ 16	4·0	56·5	3	+ 47	16
56·5	3	+ 40	13	20 2·5	5	− 62	12	2 7·5	5	− 69	14
59·5	3	− 35	12	9·0	2	+ 30	15	20·5	5	− 30	6
17 2·5	3	+149	50	12·5	3	− 30	10	36·5	5	+ 62	12
6·0	4	−153	38	21·5	3	+ 33	11	50·5	7	− 74	11
9·5	3	+ 37	12	33·5	5	− 52	10	58·0	8	+ 60	7·5
12·5	3	− 45	15	38·5	5	+ 25	5·0	3 3·5	3	− 36	12
23·5	3	+161	54	53·5	5	− 30	6·0	8·0	6	+ 42	7·0
26·5	3	−161	54	58·5	3	+ 29	9·5	48·0	20	+ 84	4·0
32·0	6	− 55	9·0								

Table VI.—Rapid Changes in W at Eskdalemuir.

Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.
h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.
14 36·0	2	+ 20	10	17 51·0	6	+ 52	8·5	21 16·5	5	+224	45
15 8·5	5	− 59	12	18 9·5	3	+ 22	7·5	23·0	8	−171	21
27·0	4	+ 31	7·5	18·0	8	− 58	7·0	30·0	6	+ 59	10
33·5	5	− 38	7·5	29·0	10	+ 84	8·5	37·0	2	+ 54	27
38·0	4	+ 32	8·0	38·0	4	− 31	7·5	38·5	1	− 35	35
54·5	7	− 80	11	43·0	2	+ 26	13	54·5	9	+ 59	6·5
16 2·5	7	+128	18	19 4·0	6	− 68	11	22 5·0	6	+ 42	7·0
7·0	2	− 35	17	15·0	6	− 66	11	17·5	7	+ 36	5·0
14·5	7	+ 80	11	25·5	3	+ 40	13	23 57·5	3	+ 22	7·5
20·5	5	−121	24	32·0	4	+ 27	6·5	1 0·5	3	+ 25	8·5
24·0	2	+ 34	17	36·0	4	− 49	12	4·5	5	− 32	6·5
35·5	3	+ 38	13	43·5	5	− 92	18	22·5	9	− 32	3·5
43·5	1	− 48	48	50·0	8	+ 58	7·0	2 17·0	12	+ 83	7·0
46·0	2	− 36	18	55·0	2	− 27	13	30·5	3	− 24	8·0
49·5	5	+ 88	18	58·5	5	+ 56	11	38·0	2	+ 28	14
56·5	3	+ 54	18	20 6·0	4	+ 23	5·5	41·0	4	− 32	8·0
17 4·0	6	−167	28	12·5	3	− 58	19	50·5	7	− 69	10
12·0	10	+203	20	24·0	2	− 35	17	59·0	4	+ 22	5·5
24·0	4	− 99	25	57·5	3	+ 22	7·5	3 5·0	6	− 29	5·0
31·0	6	− 58	9·5	21 4·5	5	−251	50	12·0	8	+ 28	3·5
38·0	8	+100	12	8·5	3	+142	47	20·5	5	+ 27	5·5
45·0	6	− 43	7·0	12·0	4	−178	44	35·5	15	− 42	3·0

Table VII.—Rapid Changes in V at Eskdalemuir.

Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.	Mean time.	Duration.	Change.	Rate.
h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.	h. m.	min.	γ.	γ/min.
15 33·5	9	+ 35	4·0	18 8·0	8	− 32	4·0	21 25·0	2	+ 22	11
54·0	8	+ 36	4·5	19 43·0	4	+ 17	4·0	35·0	2	− 19	9·5
16 1·0	6	− 28	4·5	50·0	10	− 74	7·5	38·5	5	+ 47	9·5
6·5	5	+ 32	6·5	20 2·5	9	− 37	4·0	50·5	9	− 30	3·5
20·0	8	+130	16	11·5	9	+ 26	3·0	22 1·5	5	− 43	8·5
25·5	3	− 50	17	19·0	6	− 32	5·5	21·0	10	+ 38	4·0
43·0	6	+ 35	6·0	32·5	15	− 56	3·5	2 25·5	55	−223	4·0
48·0	4	+ 56	14	21 11·0	2	− 89	44	3 12·5	25	+ 91	3·5
51·5	3	− 43	14	15·5	5	−229	46	42·5	35	+ 87	2·5
17 53·0	4	+ 19	4·5	19·5	3	+104	35				

Independent measurements were first made of the absolute times of the beginning and end of each movement. A differential measurement was then made giving the duration directly. Finally, the slope of the curve was measured with a special scale. With times measured only to the nearest minute, no very great accuracy can be claimed for the rates, but the checks mentioned above should be a sufficient protection against large errors. The probable error is naturally greatest where the time interval is least, and the large rates obtained for some of the shortest movements may thus incur suspicion. The Kew rates are all much slower than the faster Eskdalemuir rates, and the fastest of them, 27 γ per minute in H, was derived from an 8-minute interval, and so can hardly be much in error. The fastest rate for N changes at Eskdalemuir, viz. 54 γ per minute, is based on a 3-minute interval, but that interval was immediately followed by a second 3-minute interval, the change occurring during which had the same estimated rapidity. The two movements constituted an oscillation in which the to-and-fro movements were practically equal. The duration of the complete oscillation was thus a particularly convenient one to measure. If we accept six minutes for it, but suppose that an error was made in allotting equal times to the two movements, then the duration of one of them must have been over-estimated, and so its rapidity under-estimated. A somewhat similar argument applies in the case of the next most rapid N rate, viz., 50 γ per minute.

The most rapid rate for W changes in Table VI, viz. 50 γ per minute, is followed by rates of 47 γ, 44 γ, and 45 γ per minute, and these four rates arise from intervals which form between them the 17 consecutive minutes 21 h. 2 m. to 21 h. 19 m. on the 16th. The error in the total interval is unlikely to have exceeded one minute, and any redistribution of the time amongst the four sub-intervals would almost inevitably have led to at least one rate in excess of any of the four assigned.

The fastest rate for V arose from a 5-minute interval. It was a particularly favourable case for measuring the slope of the curve, as the trace was bold and the slope uniform to the eye. In the case of the longer intervals, the assigned rate must in general have been very sensibly exceeded during part of the time.

No V movement was included in Table VII, any part of which was lost through the trace going off the sheet. Several rapid movements were excluded for this reason, but none so rapid as the fastest movement given in the Table.

No rates were calculated for V changes at Kew, because little reliance could have been placed on them.

Table VIII.—Aggregates of Rapid Changes and Corresponding Mean Rates.

Element.	Rises.			Falls.			Rises and falls.		
	Aggregates.		Mean rate.	Aggregates.		Mean rate.	Aggregates.		Mean rate.
	Duration.	Movement.		Duration.	Movement.		Duration.	Movement.	
	mins.	γ.	γ/min.	mins.	γ.		mins.	γ.	γ/min.
Kew H	188	1034	5·5	175	1217	7·0	363	2251	6·2
Kew D	146	529	3·6	158	576	3·6	304	1105	3·6
Eskdalemuir N	183	2214	12·1	196	2430	12·4	379	4644	12·3
Eskdalemuir W	178	2069	11·6	156	2163	13·9	334	4232	12·7
Eskdalemuir V	137	775	5·7	138	985	7·1	275	1760	6·4

Table VIII gives particulars of the aggregates of the rises and falls included in Tables III–VII. But for the loss of trace, the time aggregate for V at Eskdalemuir would have been considerably larger. While the time aggregates for the different elements are not identical, we may fairly conclude that during the most active part of the storm changes in N and W at Eskdalemuir were roughly twice as rapid as changes in H, and $3\frac{1}{2}$ times as rapid as changes in D at Kew. The V changes at Eskdalemuir and the H changes at Kew had very similar mean rates, but the fastest V changes at Eskdalemuir were decidedly faster than the fastest H rates at Kew. The V traces were, however, of a very different type from the others, oscillations of short period being much less in evidence. Considering that the ranges of the diurnal inequalities in N and W at Eskdalemuir in December are only about 15 γ and 20 γ respectively, it is a little startling to find that during six hours of one particular "day," the mean rates of change per minute in these elements were quite 12 γ. The contributions to the aggregates of changes from the other 18 hours of the 24 would have been considerable, and the mean rate of change in N and W for the whole 24 hours could hardly have been less than 4 γ per minute.

§ 6. Any complete estimate of the expenditure of energy during a magnetic storm is probably impossible. A magnetic storm is usually accompanied by aurora and earth currents. In this particular case, according to observations made by Prof. Störmer near Christiania, the aurora extended from 100 to 400 kiloms. above the earth's surface. There is no known method of finding the intensity at any height of the electric currents producing aurora. Few stations attempt to measure earth currents, and little, if anything, is known as to the depth to which earth currents extend. Thus the energy expended in aurora, and the energy expended in earth currents, are alike inaccessible to calculation. If the ultimate source of all the terrestrial phenomena is some form of electrical discharge from the sun, the energy represented by the various phenomena in the earth and the atmosphere is probably but an insignificant fraction of the whole.

The estimate of the energy represented by the changes shown in magnetic curves is naturally a simpler problem, but even it is fraught with difficulties, practical as well as theoretical. For what has been done to give definiteness to the problem we are mainly indebted to the late Prof. Bidlingmaier.*

The formula given by Maxwell for the energy in a magnetic field when there are no electric currents is

$$(1/8\pi)\iiint(\alpha^2+\beta^2+\gamma^2)\,dx\,dy\,dz,$$

where α, β, γ represent the rectangular components of magnetic force. The integral is supposed to be taken throughout the whole of the magnetic field. It takes, moreover, as point of departure a total absence of force. In the present case we know the absolute values of the three components at a fixed point, and the sequence of changes in these components; but the intensity of the field never vanishes, the changes encountered being in fact small compared with the mean values of the elements. Suppose we confine our attention for the moment to one of the three components. The value may be regarded as given by the curve ordinate y at the instant. If y_0 be the accepted normal value, the "activity," as Bidlingmaier has called it, at the instant is

$$(1/8\pi)\,(y-y_0)^2. \tag{1}$$

* 'Veröffentlichungen des Kaiserlichen Observatoriums in Wilhelmshaven,' "Ergebnisse der Magnetischen Beobachtungen im Jahre 1911," Neue Folge, Heft 2.

What we are concerned with in practice is the mean value of the "activity" for some specified time, whether an hour, a day, or a year. Let

$$y = y_h + \eta, \tag{2}$$

where y_h is the mean value for the hour under consideration. Then, from the properties of the arithmetic mean, we have

$$\text{Mean value of } (y-y_0)^2 \text{ per hour} = (y_h-y_0)^2 + \text{mean value of } \eta^2 \text{ per hour.} \tag{3}$$

To determine the mean value of η^2, Bidlingmaier measured the curve at six-minute intervals and took $(1/10)\,\Sigma\eta^2$ as the mean value for the hour. In the present case the curve was measured at four-minute intervals, viz., at 0 4, ..., 56, 60 minutes after the hour, and

$$(1/15)\,\Sigma\eta^2 \equiv \{\tfrac{1}{2}(\eta_0{}^2+\eta_{15}{}^2)+\eta_1{}^2+\ldots+\eta_{14}{}^2\}, \tag{4}$$

was accepted as the mean. Thus $(1/8\pi)(1/15)\Sigma\eta^2$ answers to what Bidlingmaier called $A_h{}^x$ for the particular hour. The mean "activity" for the one hour is

$$A_h{}^x+(1/8\pi)(y_h-y_0)^2. \tag{5}$$

Suppose now we want the mean activity for the day. If y_d be the mean value for the day, and $y_1, \ldots, y_{24}$ the mean values for the several hours, then

$$\text{Mean value of } (y_h-y_0)^2 = (y_d-y_0)^2+(1/24)\{(y_1-y_d)^2+\ldots+(y_{24}-y_d)^2\}. \tag{6}$$

In Bidlingmaier's notation

$$(1/8\pi)\{(y_1-y_d)^2+\ldots+(y_{24}-y_d)^2\} \equiv A_d{}^h, \tag{7}$$

$$(1/8\pi)(y_d-y_0)^2 \equiv A_a{}^d. \tag{8}$$

His complete expression for the mean "activity" of the day is

$$A_h{}^x+A_d{}^h+A_a{}^d. \tag{9}$$

In this connection $A_h{}^x$ represents the arithmetic mean of the 24 individual hourly values of $A_h{}^x$. The curve ordinates are to be regarded as expressed in terms of $1\gamma\,(\equiv 1\times10^{-5}$ C.G.S.), and the unit in the "activity" results is 1×10^{-10} erg per cubic centimetre.

A difficulty at once arises as to $A_a{}^d$, viz., what to accept for the normal value y_0. Bidlingmaier apparently thought that theoretically the best plan would be to derive it from the mean value for the whole year, by assuming the rate of secular change uniform. For instance, for Wilhelmshaven in 1911, taking $+16\,\gamma$ per annum as the rate of change of H, he accepted as the value of y_0 for a day n days subsequent to July 1

$$\text{Mean for year } +(n/365)\,16\gamma.$$

His value of y_0 thus increased gradually throughout the year.

The mean value for a year is unknown until the year is completed. For the secular change it is desirable to know the mean value for the subsequent as well as the previous year. Thus the calculation of A_a^d in the way outlined above would have to be postponed to a somewhat indefinite date. Recognising this, though he gave values of A_a^d at Wilhelmshaven for all days from January 1 to June 30, 1911, the period to which he confined himself, Bidlingmaier concluded that in general the only practical course would be to omit A_a^d, *i.e.*, to accept for y_0 the mean value of the individual day. This would give for the day's mean "activity" $A_h^x + A_d^h$, the symbols having the same meaning as before.

In accepting as the best theoretical value for y_0 one based on the mean value for the year, Bidlingmaier recognised one difficulty. The mean value is not the same when we confine ourselves to quiet days as when we include all days of the year. This is especially true of H. The quiet day yearly mean would seem the more natural one to employ in the present connection, but that is a point on which opinions may differ.

Another difficulty is that an annual inequality is believed to exist in the case of all the magnetic elements. If an annual inequality really exists, it should presumably be taken into account in the present problem. These considerations support the view that the practical difficulties in the way of applying to individual days a correction A_a^d based on the mean value of the element for the year and the secular change are very serious. When we examine the results for individual days of 1911, which Bidlingmaier found at Wilhelmshaven, we find that A_a^d is by no means trifling. The following selection from Bidlingmaier's results should carry conviction on this point:—

	D alone.			H alone.			D + H.		
	A_h^x.	A_d^h.	A_a^d.	A_h^x.	A_d^h.	A_a^d.	A_h^x.	A_d^h.	A_a^d.
Mean from 6 months ...	2·55	9·72	1·15	1·73	5·40	2·05	4·28	15·12	3·20
April 9, 1911	15·5	53·8	48·8	14·9	35·6	38·2	30·4	89·4	87·0
March 20, 1911	17·4	37·5	0·0	14·6	53·5	2·0	32·0	91·0	2·0

On the average day the value of A_a^d is here about three-quarters that of A_h^x, and on some individual days A_a^d is of the same order as A_d^h. Our view as to the relative "activities" on April 9 and March 20, two of the most disturbed days of 1911, for instance, would be largely determined by the retention or neglect of A_a^d.

The adoption of Bidlingmaier's "activities" as an international scheme, in place of the present "character" figures, was under consideration before the war. The question whether a correction of the type of A_a^d should be applied ought, I think, to receive very careful consideration before any final decision is reached.

If the present method of selecting the international quiet days continues, perhaps the most satisfactory course would be to accept the mean value from the five quiet days as the normal for all days of the month. At present three months are dealt with together, and the announcement of the quiet days selected, even in pre-war times, did not appear until several months had elapsed, after the end of the quarter. No doubt matters could be accelerated by dealing with the months individually. If Bidlingmaier's scheme were substituted for the present scheme, the selection of the quiet days would entail the pre-existence of the "activity" statistics. In that event, possibly, the best course might be to accept for y_0 the mean value of the last previous day considered quiet at the particular observatory.

§ 7. In the case of a disturbance so large as that of December 16–17, 1917, a difference of 2γ or 3γ in y_0 is of minor importance. I decided to calculate two sets of "activity" results. The first set accepts for y_0 a mean derived from the "day" ending at 8 h. on December 16. The second set adopts Bidlingmaier's suggestion, and neglects A_a^d entirely, accepting for y_0 the mean value for the "day" commencing at 8 h. on December 16. For the disturbed day the mean of the 24 hourly means was taken. For the previous or quiet day, the mean was taken from measurements made at 14 h. and 20 h. on the 16th, and at 2 h. and 8 h. on the 17th. The mean of readings taken at these four hours on the average quiet day agrees very closely with the mean from the 24 hours in all the elements. With a view to possible future intercomparisons it is desirable to put on record the two sets of mean curve ordinates.

	Kew.			Eskdalemuir.		
	H.	D.	V.	N.	W.	V.
	mm.	′	mm.	mm.	mm.	mm.
Day of storm	46·2	30·4	73·2	16·8	14·9	61·5
Quiet day	44·0	39·4	69·4	21·8	15·7	55·1
Excess of disturbed day mean	+2·2	−9·0	+3·8	−5·0	−0·8	+6·4
Force equivalent	$+13\gamma$	-48γ	$+61\gamma$	-26γ	-4γ	$+28\gamma$

At Eskdalemuir the equivalents of 1 mm. of ordinate were $5{\cdot}34\,\gamma$ in N, $4{\cdot}96\,\gamma$ in W and $4{\cdot}33\,\gamma$ in V. At Kew the scale value determinations gave as the equivalents of 1 mm. of ordinate $5{\cdot}9\,\gamma$ in H, $5{\cdot}36\,\gamma$ in D, and $16{\cdot}0\,\gamma$ in V.

In presenting the results obtained from the use of the quiet day mean, I have not followed Bidlingmaier's method of obtaining a correction A_a^d. This is convenient when we confine ourselves to the mean "activity" for the whole day, but seems less appropriate when we consider individual hours. In fact it is not altogether clear how Bidlingmaier proposed to treat individual hours, except when A_a^d is neglected. In his Table XIII,* when dealing with the diurnal variation of "activity," he takes no cognizance of A_a^d. He gives, however, two sets of figures. The first set includes mean values obtained for each hour of the day from the five international quiet days of the month, the second set gives the excess over these values of the corresponding values obtained when all days of the month are included. The second set of figures are regarded by Bidlingmaier as giving the diurnal variation of disturbance. They would obviously be unaffected by any correction which was common to the all-day and quiet-day "activities."

I am doubtful whether Bidlingmaier's method of deducing a diurnal variation of disturbance is satisfactory. The differences between the curve ordinates at one and the same hour of the five quiet days of a month must represent in the main disturbance, thus it is only what is common to the five days that can well claim to represent entirely quiet conditions.

In the present case, even if I had entirely approved Bidlingmaier's procedure, it would have been impracticable to adopt it, because the selection of quiet days for December, 1917, is unlikely to be announced until midsummer, 1918.

§8. The best course seemed to be to refer each hourly measurement *directly* to the quiet day mean accepted, so that the complete mean value of the "activity" for the hour becomes

$$A_h^x + (1/8\pi)(y_h - y_0)^2. \tag{10}$$

Tables IX and X give the results thus calculated for individual hours at Kew and Eskdalemuir. The unit employed in the Tables is that used by Bidlingmaier, viz., 1×10^{-10} erg per cubic centimetre.

Bidlingmaier's notation being complicated, the following notation has been employed in the Tables and their discussion

$$\left.\begin{aligned} A_2 &\equiv (1/8\pi)(1/15)\,\Sigma\eta^2 \\ A_1 &= (1/8\pi)(y_h - y_0)^2 \\ A_1' &= (1/8\pi)(y_h - y_d)^2 \end{aligned}\right\}. \tag{11}$$

* *Loc. cit.*, p. 29.

Here η has the same meaning as in (2), while y_0 and y_h are the mean values for the days which end and commence respectively at 8 h. on December 16.

The excess in the mean value of A_1 for the day over that of A_1' is equivalent to Bidlingmaier's $A_a{}^d$. The values derived for the excess from (8), accepting the differences between y_d and y_0 given in § 7, are as follows, in terms of the unit employed in the Tables:

At Kew Observatory	6·8 in H,	92·0 in D,	146·3 in V,
At Eskdalemuir Observatory ...	27·6 in N,	1·4 in W,	30·3 in V.

All the preliminary arithmetical operations which went to the construction of the Tables were performed on differences of curve ordinates expressed in millimetres or minutes of arc, the unit 1×10^{-10} erg being introduced only in the final operation; thus an absolute identity was not to be expected between the differences between A_1 and A_1' just calculated, and those derived in Tables IX and X by taking the actual means of the 24 hourly values.

It may be well to add that a calculation was made to ascertain the order of magnitude of the corrections that would have been applied if an attempt had been made to allow for the "activity" present on quiet days. The calculations were applied to the *mean* diurnal inequalities for December as given by five selected quiet days a month.

The Kew data were for the 11 years 1890–1900 combined, the Eskdalemuir data for the single year 1914. The means of the 24 hourly values thus found for A_1' (Bidlingmaier's $A_d{}^h$) were in the usual unit:

At Kew, for **H**, 0·28; for D, 1·02; for V, 0·10; total for 3 elements 1·40;
At Eskdalemuir, for N, 0·45; for W, 0·66, for V, 0·07; total for 3 elements 1·18.

No data were available for the calculation of A_2 (or $A_h{}^x$), but the contributions from A_2 on a quiet December day must be very small. If the quiet days had been treated individually, larger mean values would have been found for A_1'. But, if we may judge from Bidlingmaier's data for January, 1911,* the whole corrections with the quiet days treated individually would not have exceeded double the figures given above, and the largest correction for any individual hour would have been only from three to four times the mean. The effect of any such corrections in Tables IX and X would have been insignificant.

The calculations were carried to the first decimal place; but it has been omitted in all the values of the total "activities," as well as in all individual cases where the value of A_2, A_1, or A_1' exceeded 100. Results are given for the two horizontal components combined, as well as for the sum from the

* *Loc. cit.*, p. 29.

Table IX.—" Activity " at Kew from 8 h. December 16 to 8 h. December 17, 1917.
(Unit 1×10^{-10} erg per cubic centimetre.)

Hour. G.M.T.	H.			D.			V.			H+D.					H+D+V.				
	A_2.	A_1.	A_1'.	A_2.	A_1.	A_1'.	A_2.	A_1.	A_1'.	A_2.	A_1.	A_1'.	A_2+A_1.	A_2+A_1'.	A_2.	A_1.	A_1'.	A_2+A_1.	A_2+A_1'.
8—9	1·3	3·2	0·6	0·2	2·5	67·0	1·5	2·5	190	1·6	5·8	67·6	7	69	3·0	8·3	257	11	260
9—10	6·2	1·4	14·4	2·4	0·2	88·0	0·8	6·7	218	8·6	1·6	102	10	111	9·4	8·3	321	18	330
10—11	7·5	4·0	0·4	0·9	2·5	67·0	0·9	6·7	218	8·4	6·5	67·3	15	76	9·3	13·3	286	23	295
11—12	5·9	46·1	17·6	0·6	4·0	57·5	2·0	1·0	174	6·5	50·0	75·1	56	82	8·5	51·0	249	60	257
12—13	1·7	5·7	0·0	1·4	6·7	48·8	1·4	1·4	116	3·1	12·5	48·8	16	52	4·5	13·9	165	18	170
13—14	5·4	57·5	24·9	0·6	0·2	84·3	1·2	7·8	84·3	6·1	57·7	109	64	115	7·3	65·5	194	73	201
14—15	12·8	224	153	2·9	1·4	116	2·2	19·3	57·5	15·8	226	269	241	285	18·0	245	327	263	345
15—16	13·8	195	129	10·4	46·1	7·8	15·1	91·8	6·7	24·3	241	137	266	162	39·4	333	144	372	183
16—17	8·9	684	555	4·7	116	1·4	12·7	518	112	13·5	800	556	813	570	36·3	1318	668	1354	704
17—18	23·3	398	301	10·3	158	7·8	2·0	1206	518	33·7	557	309	590	343	35·7	1763	827	1799	863
18—19	29·7	201	134	10·6	212	24·9	14·9	1348	603	40·3	413	159	453	199	55·2	1762	762	1817	817
19—20	43·7	9·0	31·2	6·0	555	195	4·9	1320	593	49·7	564	226	613	276	54·5	1883	819	1938	874
20—21	29·6	51·6	95·7	2·3	770	330	3·9	826	274	31·9	821	426	853	457	35·8	1648	700	1683	736
21—22	109	1220	1408	49·1	545	190	17·5	268	17·6	158	1766	1598	1924	1756	176	2033	1615	2209	1791
22—23	44·6	398	509	7·7	168	11·5	1·5	139	0·2	52·3	567	520	619	573	53·8	705	520	759	574
23—24	10·8	17·6	46·1	0·9	195	19·3	1·7	255	14·4	11·7	213	65·3	225	77	13·4	468	79·7	481	93
0—1	1·1	4·8	0·2	0·7	168	11·5	5·2	148	0·0	1·9	173	11·7	175	13	7·1	321	11·7	328	19
1—2	4·4	1·4	2·0	1·8	190	17·6	8·2	48·8	26·9	6·1	191	19·5	197	26	14·4	240	46·5	254	61
2—3	51·6	184	121	17·5	104	0·4	19·9	2·5	190	69·1	288	121	357	190	89·0	290	311	379	400
3—4	5·9	0·6	3·2	1·8	139	4·8	14·1	14·4	255	7·7	139	8·0	147	16	21·8	154	263	175	285
4—5	3·7	0·6	3·2	0·2	63·7	2·5	10·8	139	0·2	3·9	64·4	5·8	68	10	14·7	203	5·9	218	21
5—6	1·6	12·9	1·0	0·8	73·7	1·0	1·1	190	2·5	2·4	86·6	2·0	89	4	3·5	276	4·5	280	8
6—7	0·3	2·6	1·0	0·1	38·3	12·9	0·7	330	35·9	0·4	40·8	13·9	41	14	1·2	371	49·8	372	51
7—8	0·4	0·0	7·8	0·7	31·2	15·9	0·2	382	54·5	1·1	31·3	23·7	32	25	1·3	414	78·3	415	80
Mean for "day"	17·6	155	148	5·6	150	57·6	6·4	303	157	23·3	305	206	328	229	29·7	608	363	637	392

Table X.—" Activity " at Eskdalemuir from 8 h. December 16 to 8 h. December 17, 1917.
(Unit 1×10^{-10} erg per cubic centimetre.)

Hour. G.M.T.	N.			W.			V.			N+W.					N+W+V.				
	A_2.	A_1.	A_1'.	A_2.	A_1.	A_1'.	A_2.	A_1.	A_1'.	A_2.	A_1.	A_1'.	A_2+A_1.	A_2+A_1'.	A_2.	A_1.	A_1'.	A_2+A_1.	A_2+A_1'.
8—9.	2·0	3·2	51·6	0·7	0·4	0·2	0·1	1·4	46·1	2·7	3·6	51·8	6	54	2·8	5·0	97·8	8	101
9—10	19·0	12·9	2·5	4·8	2·0	0·4	0·2	4·0	57·5	23·8	14·9	2·9	39	27	24·0	18·8	60·4	43	84
10—11	19·5	5·7	7·8	1·0	0·6	3·2	0·1	6·7	63·7	20·5	6·4	11·0	27	32	20·6	13·1	74·8	34	95
11—12	1·4	4·8	9·0	4·6	15·9	24·9	0·2	7·8	70·3	6·0	20·8	33·9	27	40	6·3	28·6	104	35	110
12—13	2·8	0·6	19·3	2·6	0·2	1·4	0·2	1·0	40·8	5·4	0·8	20·7	6	26	5·5	1·8	61·5	7	67
13—14	2·4	0·0	29·0	5·0	29·0	38·3	0·0	1·0	43·4	7·3	29·1	67·3	36	75	7·3	30·1	111	37	118
14—15	7·7	9·0	70·3	14·3	163	184	1·0	0·2	26·9	22·0	172	255	194	276	23·0	172	281	195	304
15—16	30·8	19·3	0·6	18·7	84·3	100	39·0	121	29·0	49·5	104	100	153	150	88·5	224	129	313	218
16—17	64·9	31·2	121	41·6	583	623	145	1499	1098	107	615	743	721	850	252	2114	1841	2366	2093
17—18	131	694	1007	115	555	593	3·1	3514	2904	246	1249	1600	1495	1846	249	4763	4504	5012	4753
18—19	236	593	873	24·2	465	500	8·0	3305	2693	260	1058	1372	1318	1633	268	4362	4066	4631	4334
19—20	90·1	0·2	22·9	86·1	1·4	0·2	23·9	3101	2530	176	1·6	23·1	178	199	200	3103	2553	3303	2753
20—21	9·7	545	330	50·2	168	148	52·5	995	673	60·0	714	478	774	538	112	1708	1151	1821	1264
21—22	326	970	663	143	2530	2431	239	116	268	469	3500	3094	3968	3563	708	3616	3362	4324	4070
22—23	64·3	158	54·5	45·1	603	564	24·7	390	632	109	761	619	870	728	134	1151	1251	1285	1385
23—24	2·0	121	31·2	8·5	73·7	60·6	11·4	22·9	108	10·6	194	91·8	205	102	22·0	217	200	239	222
0—1	1·9	121	31·2	1·4	7·8	4·0	0·5	0·0	29·0	3·3	128	35·2	132	39	3·8	128	64·3	132	68
1—2	11·1	249	112	8·8	29·0	21·1	4·7	2·5	51·6	19·9	278	133	298	153	24·5	280	185	305	209
2—3	34·0	933	643	44·9	0·0	0·6	227	861	1220	78·9	933	643	1012	722	306	1794	1863	2099	2169
3—4	42·9	360	184	9·0	29·0	19·3	119	885	1248	51·9	389	203	441	255	171	1273	1452	1444	1623
4—5	0·9	17·6	1·0	4·6	9·0	4·8	5·5	88·0	218	5·5	26·5	5·8	32	11	11·0	115	224	126	235
5—6	0·8	43·4	2·0	1·2	0·4	0·0	1·0	12·9	84·3	2·0	43·7	2·0	46	4	3·0	56·7	86·3	60	89
6—7	0·2	17·6	1·0	0·3	2·5	0·6	0·4	1·4	46·1	0·5	20·1	1·6	21	2	0·8	21·4	47·7	22	49
7—8	0·6	19·3	0·6	0·2	6·7	3·2	0·1	0·0	29·0	0·8	26·0	3·9	27	5	0·9	26·1	32·9	27	34
Mean for "day"	45·9	205	178	26·5	223	222	37·8	622	592	72·4	429	400	501	472	110	1051	992	1161	1102

three components, because it was the only way of intercomparing the Kew and Eskdalemuir results from the horizontal plane, and because Bidlingmaier omitted vertical force.

The relative importance of A_2 and A_1' is of special interest in connection with the question of the calculation of A_2, a very laborious process when carried out strictly according to the formula. If we express A_2 as a percentage of A_2+A_1' in the case of the mean values for the day, we obtain the following results:—

At Kew, in H, 11; in D, 9; in V, 4; in H+D, 10; in H+D+V, 8;
At Eskdalemuir, in N, 21; in W, 11; in V, 6; in N+W, 15; in N+W+V, 10.

With these we may compare Bidlingmaier's mean results at Wilhelmshaven for January to June, 1911; viz., in H, 24; in D, 21; in H+D, 22. It would appear, however, that most of Bidlingmaier's results were derived not from actual measurement of $(1/10)\,\Sigma\eta^2$, but from measurements of the hourly ranges, and certain numerical relationships which he believed to hold between the value of A_h^x and the square of the corresponding hourly range. In a recent paper,* I arrived at the conclusion that Bidlingmaier's relationships were not generally satisfactory. If they had been accepted in the present case, we should unquestionably have got a higher mean value for A_2. If $A_2/(A_2+A_1')$ were always as small as the above values for Kew suggest, it could not be claimed that any very high degree of accuracy is essential in the method of calculating A_2.

Bidlingmaier found that on the average day the "activity," whether A_2, A_1, or A_1', was considerably greater for D than for H. On the present occasion, the "activity" at Kew, especially A_2 and A_1', is conspicuously larger for H than for D. The A_2 and A_1' results for N and W at Eskdalemuir differ less, which suggests that the horizontal direction in which the changes of force were most active made a fairly close approach to the magnetic meridian.

In temperate latitudes the range of the regular diurnal inequality in V is considerably less than in either horizontal component. Thus, undoubtedly, on the average day at any European station, the contribution from V will be small compared with the sums of the contributions from the horizontal components. But, on the present occasion, the contribution from V to A_2+A_1 or A_2+A_1' at Kew is nearly equal to the combined contributions from H and D, while at Eskdalemuir the contribution from V considerably exceeds that from N and W combined. This shows that the question of the

* 'Terrestrial Magnetism and Atmospheric Electricity,' vol. 22, p. 57 (1917).

exclusion of V data from an international scheme possesses an importance which is not suggested by a study confined to ordinary days.

§ 9. It is interesting to note, confining ourselves to the results for $H+D$ that the mean value of A_2+A_1' at Kew on December 16–17 was 11·8 times that of the average day from January 1 to June 30, 1911, at Wilhelmshaven, and 15 times that of the average January day. Also, the mean value of A_2+A_1' at Kew between 21 h. and 22 h. on December 16 was 52 times that of the corresponding hour of the average January day at Wilhelmshaven. It is obvious that, when we confine ourselves to data from a single year, a few highly disturbed days might exercise a swamping influence in statistics as to the annual or diurnal variation of magnetic "activity."

During the earlier part of the storm the "activity" at Kew was not inferior to that at Eskdalemuir, but after 16 h. on the 16th the Eskdalemuir "activity" was much the greater. At both stations, so far as the horizontal field is concerned, 21 h.–22 h. on the 16th was decidedly the most disturbed hour. This was the time at which aurora was brightest, according to reports from meteorological stations, several Scottish stations reporting a corona not far from the zenith. Kew and Eskdalemuir also agree in showing a maximum of "activity" in V between 17 h. and 20 h. on the 16th, and a marked recrudescence of "activity" between 2 h. and 4 h. on the 17th, which was presumably associated with auroral streamers observed at Southport between $2\frac{1}{4}$ h. and $2\frac{3}{4}$ h.

§ 10. In view of the great labour involved in applying Bidlingmaier's scheme, and the various uncertainties, I suggested in 'Terrestrial Magnetism' (*loc. cit.*) that a much simpler scheme, if applied at the numerous observatories which participate in the present international scheme, might suffice for the magnetic classification of days. The suggestion was that, if R_h, R_d, R_v represent the absolute ranges in the three magnetic elements, $R_h^2+R_d^2+R_v^2$, or, if the horizontal field only is considered, $R_h^2+R_d^2$ might be accepted as a measure of the mean "activity" for the day. If the horizontal components recorded were N and W, then $R_n^2+R_w^2$ would take the place of $R_h^2+R_d^2$.

I have compared the mean monthly values of the "activity" at Wilhelmshaven for three of the six months dealt with by Bidlingmaier, with the corresponding arithmetic means of the squares of the Kew daily ranges. The following were the results obtained :—

	January.	March.	May.	Mean.
$\dfrac{\text{Mean value of } A_2+A_1' \text{ at Wilhelmshaven}}{\text{Mean value of } R_h^2+R_d^2 \text{ at Kew}}$	0·000196	0·000224	0·000194	0·000205
$\dfrac{\text{Mean value of } A_2+A_1 \text{ at Wilhelmshaven}}{\text{Mean value of } R_h^2+R_d^2 \text{ at Kew}}$	0·000221	0·000251	0·000235	0·000236

Taking now the "activities" and ranges found for December 16–17, we get the following results:—

At Kew—

$$(\text{Mean } A_2+A_1' \text{ for } H+D)/(R_h^2+R_d^2) = 0{\cdot}000113,$$

$$(\text{Mean } A_2+A_1' \text{ for } H+D+V)/(R_h^2+R_d^2+R_v^2) = 0{\cdot}000146,$$

$$(\text{Mean } A_2+A_1 \text{ for } H+D)/(R_h^2+R_d^2) = 0{\cdot}000162,$$

$$(\text{Mean } A_2+A_1 \text{ for } H+D+V)/(R_h^2+R_d^2+R_v^2) = 0{\cdot}000237.$$

At Eskdalemuir—

$$(\text{Mean } A_2+A_1' \text{ for } N+W)/(R_n^2+R_w^2) = 0{\cdot}000065,$$

$$(\text{Mean } A_2+A_1' \text{ for } N+W+V)/(R_n^2+R_w^2+R_v^2) = 0{\cdot}000106,$$

$$(\text{Mean } A_2+A_1 \text{ for } N+W)/(R_n^2+R_w^2) = 0{\cdot}000069,$$

$$(\text{Mean } A_2+A_1 \text{ for } N+W+V)/(R_n^2+R_w^2+R_v^2) = 0{\cdot}000112.$$

The difference between the above figures for the two stations arises from A_1' and A_1. If we confined ourselves to A_2 we should get in place of the above ratios: For the horizontal components, 0·000012 at Kew, 0·000010 at Eskdalemuir; for all three components, 0·000011 at both places.

The means of the absolute daily ranges for the whole year 1914 were

At Kew, 44γ in H, and 55γ in D;

At Eskdalemuir, 60γ in N, and 58γ in W.

This makes

$$(R_n^2+R_w^2)/(R_h^2+R_d^2) = 1{\cdot}40.$$

Wilhelmshaven being very nearly midway as regards latitude between Kew and Eskdalemuir, the absolute ranges at Wilhelmshaven probably exceed those at Kew. Thus if we had employed Wilhelmshaven instead of Kew ranges for comparison with the Wilhelmshaven "activities" of 1911, we should most likely have got smaller values than those obtained above for the ratios.

The scheme which I suggested does not postulate that the ratio borne by "activities" to squares of ranges should be the same for all stations. It might well be different at a quiet station like Helwan, and at a disturbed station like Sitka. Everything considered, however, it is probable that on any scheme which assumed "activity" proportional to the square of the absolute range, the mean "activity" for December 16–17 would have been sensibly over-estimated at Kew, and considerably over-estimated at Eskdalemuir. On the other hand, the large difference between the values of A_1 and A_1' in Table IX, and the uncertainty this implies, shows that even with the elaborate procedure entailed by Bidlingmaier's full scheme, there is a large probable error at individual stations.

Under these circumstances a simple scheme, even if admittedly imperfect, deserves consideration.

A Method of Avoiding Collision at Sea.

By J. Joly, Sc.D., F.R.S., a Commissioner of Irish Lights.

Part I.

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The following method of avoiding collision at sea depends on the use of synchronised signals, transmitted in different media. Such signals, travelling at different rates, enable the distance of their source to be inferred by observation of the gain in time of the faster upon the slower travelling signal. Thus, if signals be simultaneously emitted by wireless and by submarine bell (or Fessenden oscillator), the former being transmitted with practically infinite velocity, the latter arrive with a lag which is the time the submarine sound requires to traverse the intervening medium. The rate of propagation of sound in water being closely 4800 feet per second, the lag is 0·62 second for one-half sea-mile.

In practice the signals may be so ordered as to dispense with the stop-watch or chronograph. This is accomplished by sending out the wireless ticks in groups of, say, 20 "dots" spaced to intervals of 0·6 second. The stroke of the bell precedes the first of these dots by one of these intervals. Thus, when the sailor is half mile from the source he hears the first wireless dot along with the bell stroke. If he is 1 mile distant the bell stroke comes in with the second dot, and so on. He has, in fact, only to count up the dots till he hears the bell, and the number of the dot coincident with the bell is the number of half sea-miles intervening between his ship and the source of the signals. It is possible to estimate the quarter mile by noting a want