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When there is a perfectly conducting sphere of radius b concentric with the sphere of radius a the value of the ratio $\psi/\overline{\psi}$ cannot be deduced at a point on the surface of the sphere of radius b which is on or near to a caustic as there is a diffraction effect due to the conducting sphere which is not negligible at such a point, but it can be inferred that the value of the ratio lies between four times the above value and the above value.

On the Variability of the Quiet-Day Diurnal Magnetic Variation at Eskdalemuir and Greenwich.

By S. Chapman, F.R.S., and J. M. Stagg, M.A., B.Sc.

(Received December 6, 1928.)

Introduction.

§ 1. The solar diurnal magnetic variation on quiet days (conveniently denoted by S_q) undergoes more or less *regular* changes of two kinds: one, affecting both type and amplitude, in the course of each year—an annual variation; the other, affecting chiefly the amplitude, in the course of the sunspot cycle—this will be termed the solar-cyclic change. In addition, it undergoes irregular or fortuitous changes from day to day. These have scarcely been studied as yet, and there is no precise systematic information available concerning them. They are of considerable interest owing to their probable connection with day-to-day changes in the sun's ultra-violet radiation. We therefore propose to examine them systematically, especially with reference to their similarity at different stations. In this paper we confine ourselves to the records for Eskdalemuir and Greenwich, the respective geographical co-ordinates being :—

	Latitude.		Longitude.	
	°	′	°	′
Eskdalemuir	55	19 N	3	12 W
Greenwich	51	28 N	0	0

The material used consists of the daily ranges, in all three magnetic elements, on 819 very quiet days. It is shown that the ranges suffice for the comparison of different days, since (§ 15) the irregular changes in S_q, at any particular season and solar epoch, affect almost solely the amplitude of the daily variation, while the type, that is, the form of the curves representing the diurnal inequalities, remains constant.

The fortuitous percentage departures of the daily ranges from their normal value at any season and solar epoch are found to be considerable, often amounting to $\pm$ 25 per cent., while much larger departures are not uncommon (§ 14).

On the average the departures from the normal ranges are of the same sign, on any day, for all three elements at both observatories, though there are many individual exceptions to this rule. The correlation between the departures for the same element at the two observatories is much closer than that between the departures of different elements at the same observatory (§§ 16, 17).

Besides these main results, some interesting and unexpected correlations have been discovered between the fortuitous variations of S_q and of the non-cyclic changes in north and vertical force, and also between these and the departures of the daily mean values of the magnetic elements from their monthly means.

Symbols of Abbreviation.

§ 2. The discussion is facilitated by the use of the following symbols :—

E = Eskdalemuir. G = Greenwich.

w = winter (November–February).

s = summer (May-August).

e = the remaining " equinoctial " months.

y = year.

N = north component of magnetic force.

Z = downward vertical component.

H = horizontal component.

W = west component, at E, or west declination, at G, the changes in the latter case being measured in force units.

$\cap$ = sunspot maximum.

$\cup$ = sunspot minimum.

R = the range, on any day, in the diurnal inequality of a magnetic element, as expressed by hourly means or hourly values ; the distinction is unimportant in the case of the really quiet days here considered.

$\bar{R}$ = the mean value of R for any assigned interval or group of days.

R_n = the adopted normal value of R at any particular date (*cf.* § 12).

ΔR = the percentage departure of the actual value of R on any day from the normal value for that date (*cf.* § 13).

α = the non-cyclic variation in any element on any day, *i.e.*, the (positive or negative) excess of its value at the end of the day (24^h) over its initial value (at 0^h).

D = the (positive or negative) excess of the daily mean value of any element over the mean value for the same calendar month.

The notations $\bar{\alpha}$, α_n, $\Delta\alpha$, $\bar{D}$, D_n, ΔD will be used in the same sense relative to α and D as for R.

Unit of Force.

§ 3. The unit of force used throughout the paper is 1γ, or 10^{-5} gauss.

Period and Grouping of the Years considered.

§ 4. The data used relate to the well-developed sunspot-cycle 1913–23. The mean sunspot number for each year is given in Table I, row 2. Sunspot maximum occurred in 1917, and sunspot minima in 1913 and 1923.

In parts of our work we have divided the whole set of 11 years into the two following groups of years of greater and less sunspottedness. Data referring to these groups will be distinguished by the symbols ∩, ∪.

Group ∩; 5 years; 1915–19; mean sunspot number, 70·5.

Group ∪; 6 years; 1913, 1914, 1920–23; mean sunspot number, 15·8.

The Choice of Quiet Days.

§ 5. The choice of quiet days was based on the international magnetic character figures, which give an indication of the degree of disturbance existing on each Greenwich day from midnight to midnight. The figures range from 0·0, representing extreme quiet, to a maximum value 2·0. In investigations relating to quiet days it is customary to use the five quiet days per month chosen under the international scheme, but in some months these may include days that are not quite quiet. In order to ensure that the days we used should be as quiet as possible, we confined ourselves to those having character figures 0·0 or 0·1, however many or however few there might be in any particular month.

In some parts of our work we have separately considered the two classes of day, 0·0 and 0·1, but the results show that there is very little systematic difference between them as regards S_q-variability. Hence in general we have used the two sets of days in combination without distinction.

The total number of quiet days used was 819, one-third being of character figure 0·0 and two-thirds 0·1. The average number per month was 2·05 of character 0·0, and 4·15 of character 0·1. The average total number of quiet days used per month, 6·2, exceeds the monthly number of international quiet days (5), but this excess proceeds from the minimum solar years ($\cup$), when an average of 6·76 days of character 0·0 and 0·1 were available; in the maximum solar years ($\cap$) the average number of days used per month was 5·53. It is not improbable that the days here considered are slightly quieter than the international days, or, at least, include fewer that are slightly disturbed.

Tables I and II show the distribution of the 0·0 and 0·1 days between the different years, and between the calendar months and seasons; in the latter case the distribution of 0·0 and 0·1 days combined is given for the $\cap$ and $\cup$ years separately.

Description of the Original Data.

§ 6. The ranges in the hourly inequalities of the three magnetic elements at E and G for the selected quiet days were abstracted from the published annual records of the magnetic observations at these stations.

The elements registered at E throughout the 11 years covered by the investigation were N, W and Z, while at G they were west declination (D), Z and N, except that in 1913 and 1914 data for H instead of N were given. With a mean declination of just over 15° for 1913 and 1914 the disparity between the daily ranges of horizontal force on quiet days measured along the geographical and magnetic meridians was considered sufficiently small to allow the H ranges to be treated as those from a north force instrument.

The individual declination ranges in minutes of arc were converted into force units by means of the relation

$$\Delta \mathrm{D}\ (\gamma) = \Delta \mathrm{D}\ (\text{mins. of arc}) \times \mathrm{H} \times \text{arc } 1',$$

using mean values of H for each of the 11 years. The secular change in H at G resulted in a decrease from 18530 γ in 1913 to 18432 in 1923, so that the equivalent of 1′ change in D fell from 5·39 γ in the former to 5·36 γ in the latter year. The ranges of west D (so converted) were treated as ranges of west force.

At E the published hourly values are means for hourly intervals centred at the exact hours of Greenwich mean time; at G during 1913 and 1914 small irregularities in the magnetograms were first smoothed out, and then instantaneous hourly values at each exact hour tabulated. Subsequent to 1914, mean values for the intervals between successive exact hours of Greenwich mean time have been published.

Examination of the actual E magnetograms for a number of quiet days for which the tabulated ranges appeared to be in some way irregular, showed that though complete absence of small perturbations was rare, the use of hourly means would in all but a few cases give values which smoothed out the effects of these perturbations. The preliminary pencil smoothing of the curves for the G data of 1913–14 would have a similar effect.

Temperature corrections for the records from N (or H) and W variometers are accurately determinable, and D requires no temperature correction, but the method of registration of Z is such that the necessary correction may be both large and inaccurate. At both E and G the corrections for the vertical force variometers are large, 26 γ for 1° C. throughout the period after June, 1913, at E (no Z data are published for the earlier months of that year), and 17 γ up to February, 1917, at G. After February, 1917, a new variometer, subsequently compensated for temperature changes as an additional safeguard, was run at G in a room whose temperature was controlled by a thermostat.

Since the amplitude of the diurnal inequality is least in the winter months, accidental inaccuracies arising from faulty temperature correction are proportionally greater than in the other seasons (see § 14).

The 11-*year Mean Values of* R.

§ 7. In Table I the mean ranges are given for each year (all months), and in Table II for each calendar month and season (all years), for the 0·0 and 0·1 days combined. The results are illustrated in figs. 1 and 2 (*a*); in every case there is a considerable annual and solar-cyclic variation (§§ 8, 9). In forming

Table I.—Mean Annual Sunspot Number, Number of Quiet Days used, and Mean Range, for each Year, 1913–23.

——	1913.	1914.	1915.	1916.	1917.	1918.	1919.	1920.	1921.	1922.	1923.	Mean.
Sunspot number	1·4	9·6	47·4	57·1	103·9	80·6	63·6	37·6	26·1	14·2	5·8	40·7
Number of 0·0 days	36	36	34	20	22	18	19	20	16	23	27	25
Number of 0·1 days	59	61	58	39	44	41	37	56	45	37	71	50
All days	95	97	92	59	66	59	56	76	61	60	98	74
$\bar{R}$ (EN)	29·7	33·4	36·5	42·7	48·3	44·9	43·5	41·7	36·5	33·4	29·7	38·2
$\bar{R}$ (GN)	28·3	31·1	30·7	36·9	40·9	38·4	36·0	36·7	31·5	29·9	26·2	33·3
$\bar{R}$ (EW)	33·5	34·8	38·4	48·2	50·1	45·6	43·1	42·9	41·0	36·4	32·5	40·6
$\bar{R}$ (GW)	34·8	35·2	41·8	50·8	55·0	50·0	47·4	45·8	41·9	37·2	34·6	43·1
$\bar{R}$ (EZ)........................	12·5	13·8	14·9	15·5	17·5	17·4	17·4	16·2	14·4	14·5	13·2	15·2
$\bar{R}$ (GZ)........................	23·0	19·9	20·1	20·1	25·4	20·0	21·3	20·0	17·5	18·1	17·7	20·3

these means equal weight has been given to the mean range for each individual calendar month, irrespective of the number of 0·0 and 0·1 days included in it. Had this not been done, the $\sqcup$ years, which contribute a larger proportion of the days than the $\sqcap$ years, would have had undue weight, and the means would have been too small.

The ranges in N and W are of the same order of magnitude, the 11-year means, for E and G combined, being 35·8 and 41·9 ; the corresponding value for Z, 17·8, is much smaller.

The 11-year mean values are less at E than at G, in Z and W, and greater in N. The differences E − G, expressed as percentages of the mean $\frac{1}{2}$(E + G), are as follows :—

Values of $100\,\{R\,(E) - R\,(G)\}/\frac{1}{2}\,\{R\,(E) + R\,(G)\}$

N.	W.	Z.
13·7	−6·0	−28·7

The percentage difference is notably greater for Z than for N and W, and the sign of the difference in Z is of special interest. On disturbed days the ranges are greater at E than at G in all elements, but especially in Z ; whereas we here see that on quiet days R (Z E) is *less* than R (Z G), and by an amount proportionally much greater than for N and W.

S_q in Z and W changes sign at or near the equator, and R (Z), R (W) are there small, and increase polewards. At the poles R (Z) must be zero, at least in so far as S_q there depends on local time ; hence R (Z) should have a maximum between the equator and the pole. It appears from the above that this maximum occurs south of E.

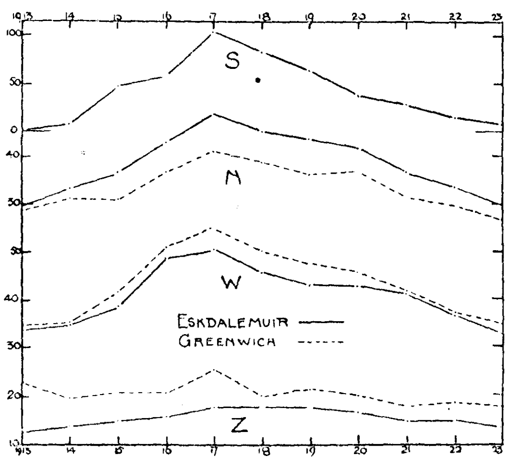

Fig. 1.—Mean Annual Sunspot Numbers (S) and Inequality Ranges, 1913–23.

There is no such limitation of N and W at the poles, but Table I suggests that R (W) also has a maximum south of E.

S_q in N changes sign in latitude 30° or 35°, and R (N) increases from this latitude towards both the equator and the pole. The poleward increase would appear to extend at least as far north as E.

The Solar-cyclic Variation.

§ 8. From Table I and fig. 1 it appears that the solar-cyclic variation is most regular in N and W, being an unbroken ascent to and descent from sunspot maximum, except for a slight irregularity in R (N, G) in 1919. In Z the variation of R is regular at E, but the progression in the Greenwich values of R (Z) is anomalous.

The increase from sunspot minimum (the mean of 1913 and 1923) to sunspot maximum (1917), expressed as a percentage of the 11-year mean R, is as follows :—

Values of $100\{R(\sqcap) - R(\sqcup)\}/\bar{R}$.

R (N, E)	R (N, G)	R (W, E)	R (W, G)	R (Z, E)	R (Z, G)	Mean
48·7	41·0	42·1	47·1	30·6	24·9	39·1

The Annual Variation.

§ 9. Table II and fig. 2 show that the annual variation in the S_q range is, on the whole, a regular rise and fall between a sharp winter minimum and a broad summer maximum, or series of maxima. The curves show some slight irregularities in this progression, the chief being a dip in R (N) and R (W) in May ; this seems to be real, since it appears in almost every subdivision of the data. The maximum in R (Z) occurs earlier than in R (N) and R (W), by about a month.

Figs. 2 (*c*), 2 (*d*), for the $\sqcap$ and $\sqcup$ groups of years, suggest that there is no significant change in the type of the annual variation from sunspot maximum to minimum. The amplitude of the annual variation is greater for the $\sqcap$ group, but the changes expressed as percentages of the annual mean value for the group of years considered are about the same in the two cases. This is illustrated by the following table, which gives these percentages, corresponding to the difference between the mean ranges for summer (§ 2) and the mean range for the month of minimum (December in all cases save for R (Z, G, $\sqcap$), when it was January) ; the mean for the four summer months was chosen instead of that for the actual month of maximum range, because of the somewhat irregular progression of $\bar{R}$ during this season.

Table II.—Number of Quiet Days (character 0·0 and 0·1) used in each Calendar Month and Season, and Mean Values of R for 0·0 and 0·1 Days combined.

Group of years.	Type of day.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Summer.	Equinox.	Winter.	Year.	Average for month.
All	0·0	23	16	14	20	27	28	29	19	22	22	22	29	103	78	90	271	2·05
All	0·1	43	54	42	51	42	47	52	37	34	38	60	48	178	165	205	548	4·15
All	0·0+0·1	66	70	56	71	69	75	81	56	56	60	82	77	281	243	295	819	6·20
∩	0·0+0·1	32	28	23	27	20	39	33	24	27	27	27	25	116	104	112	332	5·53
∪	0·0+0·1	34	42	33	44	49	36	48	32	29	33	55	52	165	139	183	487	6·76
Element	Station.	$\bar{R}$ on 0·0 and 0·1 days combined (all years).															Year.	
N	E	20·1	24·4	37·9	49·6	48·3	52·5	49·8	49·8	47·2	37·4	23·5	16·5	50·1	43·0	21·1	38·1	
N	G	22·6	23·9	33·5	40·1	37·4	42·9	40·8	42·9	39·6	34·9	23·2	17·9	41·0	37·0	21·9	33·3	
W	E	21·3	25·5	40·4	50·3	52·1	57·5	57·8	52·2	45·3	38·3	26·3	18·2	54·9	43·6	22·8	40·4	
W	G	23·8	28·0	44·8	55·6	55·1	58·4	58·1	55·0	48·6	42·3	29·0	19·2	56·7	47·8	25·0	43·2	
Z	E	7·7	10·3	15·7	19·9	23·2	21·4	22·8	20·0	15·7	11·6	7·9	7·1	21·8	15·7	8·3	15·3	
Z	G	11·4	14·9	21·0	27·8	29·2	26·8	26·1	23·4	20·4	16·2	13·0	11·0	26·4	21·3	12·6	20·1	

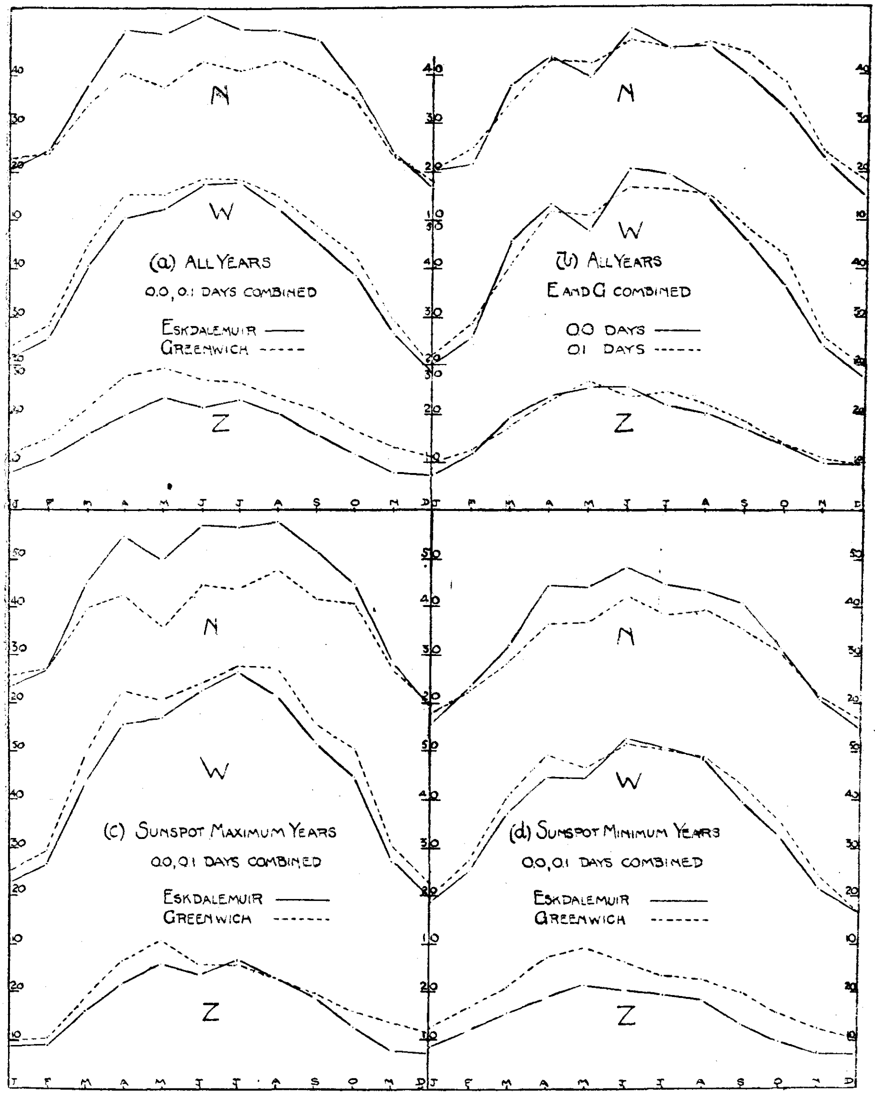

FIG. 2.—ANNUAL VARIATION OF INEQUALITY RANGES AT ESKDALEMUIR AND GREENWICH

Percentage increase of range from winter to summer :—

	R (NE)	R (NG)	R (WE)	R (WG)	R (ZE)	R (ZG)	Mean
∩	84	65	93	86	107	84	86
∪	91	74	89	85	89	76	83

It appears from this table that the percentage annual variation is about twice as great as the percentage solar-cyclic variation, the excess being most notable in the case of Z.

Comparison between 0·0 *and* 0·1 *Days.*

§ 10. An examination of the mean ranges for the 0·0 and 0·1 days separately shows that the systematic differences between them are small. The difference R (0·1) — R (0·0), expressed as a percentage of the mean range for the combined set of days, is given for each season and for the year in the following table; the differences for the year are given separately for the ∩ and ∪ groups.

——	Summer.	Equinox.	Winter.	Year.	
				∩	∪
R (NE)	2	5	11	6	7
R (NG)	−1	3	8	6	2
R (WE)	−1	2	10	1	6
R (WG)	−2	0	7	1	3
R (ZE)	7	2	18	5	10
R (ZG)	1	−3	4	−1	0
Mean	1·0	1·5	9·7	3·0	4·7

While the most important feature of the table is the smallness of the percentage difference in nearly all cases, the effect of the slight amount of disturbance which distinguishes the 0·1 days from the 0·0 days is apparent in the table, particularly in winter; though disturbance is most frequent at the equinoxes, the winter values of R are so small that even a slight amount of disturbance produces an appreciable percentage change in R. Similarly, though disturbance is more common in ∩ than in ∪ years, the smaller values of R in the latter years renders $\{R(0\cdot1) - R(0\cdot0)\}/\bar{R}$ greater in ∪ years. The greater degree of disturbance experienced at Eskdalemuir than at Greenwich is also manifested in the table, particularly as regards the vertical force Z (*cf.* § 7).

The annual variations for the 0·0 and 0·1 days separately, for all years and for E and G combined, are shown in fig. 2 (*b*), which again illustrates how small are the differences between the two sets of days. An unexpected feature shown by the curves is that the above excess of R (0·1) over R (0·0) for the equinoctial season is due to September and October, the difference being actually reversed for the two vernal equinoctial months.

The differences between R (0·1) and R (0·0) are on the whole so small that in general no distinction is made between the two sets of days in the further discussion.

Examples of Fortuitous Variations.

§ 11. As stated in § 1, there are irregular variations of S_q in addition to the regular annual and solar-cyclic changes. These fortuitous variations can be illustrated most effectively by quoting a number of cases where R in one or more elements undergoes a marked change on *successive* 0·0 days, for here there can be no possibility of ascribing the change to the regular variation of S_q.

Table III.

—	Eskdalemuir.			Greenwich.		
	R (N).	R (W).	R (Z).	R (N).	R (W).	R (Z).
1919. February 10	42	27	8	39	65	13
1919. February 11	23	24	6	21	50	5
1922. May 29	63	44	14	53	87	26
1922. May 30	44	52	19	39	96	25
1920. July 28	44	52	16	38	98	24
1920. July 29	61	58	22	54	115	29
1923. September 21	29	31	12	24	54	15
1923. September 22	43	46	14	43	95	15
1913. October 2	37	47	10	35	94	26
1913. October 3	29	36	8	37	66	33
1913. November 14	20	22	4	14	43	18
1913. November 15	10	19	5	9	48	18
1913. November 16	21	23	4	15	49	16

Many other examples might be given, but these suffice to demonstrate the existence of irregular changes ; they also show that the three elements do not always vary in parallel at the same observatory, nor does the same element always vary alike at the two observatories.

The magnetograms for Eskdalemuir were examined to see whether slight irregularities in the curves could account for the occasional instances of opposite variations in R (N) and R (W) on successive 0·0 and 0·1 days. A few such cases were found, particularly on 0·1 days, but on the whole the examination showed that the ranges on these quiet days are seldom affected by disturbance, which, being slight, can alter the range only if it occurs near the turning points on the curve.

The Normal Range on any Date.

§ 12. In order to investigate the fortuitous changes in S_q it is necessary to allow for the regular annual and solar-cyclic variations. The annual variation is the more important of the two, and produces considerable percentage changes in R even in the course of a single month, especially near the equinoxes.

It was decided to construct graphs which should give the supposed normal or standard value of R on each individual date throughout the 11 years ; this was done in the following way. It was assumed that the annual variation of R (for any element at E or G) in any year was proportional to the 11-year mean annual variation given in Table II and illustrated in fig. 2 (*a*) ; the constant of proportionality was taken to be the ratio of the mean R for that year to the 11-year mean, as given in Table I. For example, the annual variation of R (NE) in 1913 was taken to be that given for R (NE) in Table II, multiplied by 29·7/38·2 or 0·78 ; the factors used, based in this way on Table I, are given in the following Table IV.

Table IV.—Ratios of $\bar{R}$ for each Year to the corresponding 11-year Mean of R.

	—	1913.	1914.	1915.	1916.	1917.	1918.	1919.	1920.	1921.	1922.	1923.
Eskdalemuir	N	0·78	0·87	0·96	1·12	1·26	1·18	1·14	1·09	0·96	0·87	0·78
	W	0·83	0·86	0·95	1·19	1·23	1·12	1·06	1·06	1·01	0·90	0·80
	Z	0·82	0·91	0·98	1·02	1·15	1·14	1·14	1·07	0·95	0·95	0·87
Greenwich	N	0·85	0·93	0·92	1·11	1·23	1·15	1·08	1·10	0·95	0·90	0·79
	W	0·81	0·82	0·97	1·18	1·28	1·16	1·10	1·06	0·97	0·86	0·80
	Z	1·13	0·98	0·99	0·99	1·25	0·99	1·05	0·99	0·86	0·89	0·87

The monthly mean values of R thus derived for each calendar month of any year were taken to refer to the middle of that month, and a graph was constructed for each element and each year, having R as ordinates and daily epoch in the year as abscissæ, by joining the ends of the ordinates for the middle points of successive months by straight lines. That is, R was supposed to vary uniformly from the middle of one month to the middle of the next. The ordinate on any date was taken as defining the normal value of R on that date ; this can only be an approximation to the truth, but the method at least gives a value of R which takes into account much the greater part both of the annual and solar-cyclic variation of R.

The Percentage Departures of R.

§ 13. As described in § 12, the normal value (R_n) of R was determined for each of the elements at E and G, on each of the 819 quiet days used. In general R_n for any day differed from the actual value of R for the day. The difference $R - R_n$ was expressed as a percentage departure (ΔR), in terms of the mean value ($\bar{R}$) of R for the corresponding month : the precise definition of ΔR is therefore

$$\Delta R = 100\ (R - R_n)/\bar{R}.$$

Tables of ΔR, thus determined, formed the basis of the discussion of the fortuitous changes of S_q.

The Distribution of the Percentage Departures ΔR.

§ 14. In order to examine how the percentage departures ΔR were distributed, a table was constructed showing, for each set of values of ΔR, how many occurred in each of a series of intervals 5 units wide, spaced consecutively on either side of a middle interval centred at $\Delta R = 0$; any such interval can be specified by its central value of ΔR, which is always an integral multiple of 5.

The number of occurrences of ΔR in each interval was then divided by the total number (n) of values in the set under consideration ; the result, multiplied by 1000, is termed the *frequency f* of occurrence of ΔR for that interval. The sum of the frequencies for all the intervals is therefore 1000.

The values of f are given, to the nearest unit, in Table V ; the three main sections refer to N, W, Z ; in each of these are given first the E and then the G values for the year and the three seasons, from 0·0 to 0·1 days combined ; the remaining rows refer to the E and G data combined, for the whole year, and give the frequencies for the $\cap$ and $\cup$ years separately, and for the 0·0 and 0·1 days separately. The table extends from the interval centred at $\Delta R = -75$ per cent. (corresponding to $R = \frac{1}{4}R_n$) to $\Delta R = +100$ per cent. (corresponding to $R = 2R_n$). Negative departures numerically greater than 100 per cent. (or, more strictly, $100R_n/\bar{R}$) are impossible by virtue of the definition of ΔR ; actually the extreme negative departures are those given in the column $\Delta R = -75$. The positive departures are not subject to any such limit, and actually there were 13 days, all of character 0·1, for which, in one or other element, $\Delta R > 100$; these are not included in Table V. None of these refers to N, so that the first section of Table V is complete. One case refers to W, namely, ΔR (WE) = 120 on January 23, 1922 ; but this anomalous W range arises because a disturbance began during the last hour (24) of the day (the following day was of character 1·8) ; if this last hourly value were ignored, ΔR (WE) for this day would be reduced to 63, the day being really one of large range in WE, though not so abnormal as the uncorrected value of ΔR (WE) would suggest. The other 12 cases relate to Z, 10 at E and 2 at G ; of these 9 (7 E, 2 G) occur in winter, 2 (E) in e, and 1 (E) in s ; 9 are in $\cup$ years, and 3 in $\cap$; they are probably partly due to small perturbations, which affect ΔR specially in winter and $\cup$ years, when R is small.

The frequency distributions in Table V are illustrated in fig. 3. Inspection

Table V.—Frequency Distribution of ΔR (§ 14).

Component and observatory.	Group.	No. of days.	Negative values of ΔR.															0	Positive values of ΔR.																			
			75	70	65	60	55	50	45	0	35	30	25	20	15	10	5		5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
NE	*y*	810		1	2	6	6	12	9	16	16	40	42	36	84	96	95	93	99	74	68	65	47	26	16	7	10	9	7	4	1	4	4	3	1		1	
	s	280									7	29	50	29	111	107	121	125	93	79	75	61	43	32	14	7	11	3						3				
	e	241								8	9	21	37	38	83	104	83	112	112	91	83	95	62	21	9	4		12	12	4								
	w	289		3	7	17	17	35	24	38	31	66	38	42	59	80	80	45	93	56	48	45	38	24	24	11	17	10	10	7	4	10	10	4	4		3	
NG	*y*	809		1	1	4	6	12	20	20	21	52	52	63	64	83	93	64	74	78	63	53	45	30	25	24	9	4	10	6	6	7	6	2	1	1		
	s	277						7	11	7	18	36	61	43	98	87	101	72	69	115	83	43	36	25	18	18	7	11	11	4	11	4		4				
	e	241						8	12	13	4	33	58	62	54	83	108	87	96	62	95	83	42	21	21	17	13		4	4	4	4	12					
	w	291		3	3	10	17	21	34	38	38	83	38	83	41	79	72	38	62	55	17	38	55	41	35	34	7		14	10	3	14	7	4	3	3		
E + G......	∩	653				3	5	8	9	12	20	49	35	50	81	92	95	92	90	86	60	51	60	24	18	12	6		11	3	5	3	5	5		1		
	∪	966		2	3	6	7	16	18	22	18	43	55	49	69	88	93	70	78	69	69	65	36	30	22	18	12	10	7	6	3	7	5	1	2		1	
	0·0	536			5	6	9	17	19	15	19	73	50	56	91	76	95	75	80	60	58	54	47	32	19	11	7	7	6	2	2	4	3	2				
	0·1	1083		2		5	5	10	12	19	18	32	45	46	66	96	93	80	90	84	69	72	45	26	21	18	10	6	10	6	5	6	6	3	2	1	1	
WE	*y*	807	2		1	6	4	1	11	11	22	35	56	72	67	93	98	110	82	73	61	53	32	31	22	24	4	11	5	3	4	3		1	1			
	s	278								7	4	25	36	72	72	90	130	130	93	97	87	50	25	36	26	11		7	3									
	e	240								4	4	21	50	79	37	113	96	125	88	79	75	71	42	50	21	29	0	8	0	4	4							
	w	289	7		4	17	10	4	31	21	55	55	80	66	87	80	69	80	66	45	24	41	31	10	21	31	10	17	10	4	7	7		4	3			
WG	*y*	812		1	2	1	4	6	14	11	11	36	49	62	81	85	97	108	92	85	65	51	49	25	27	9	12	5	3	4	1	1			3			
	s	278									4	43	32	75	83	90	83	130	112	111	68	65	61	7	14	11	4	4	3									
	e	243			4						8	4	45	41	78	78	78	157	115	95	66	66	58	50	33	8	12			4								
	w	291		3	3	4	10	17	38	31	21	55	69	65	82	86	127	48	55	52	62	24	31	21	34	7	21	10	4	7	3	3			7			
E + G......	∩	656				3	2	3	11	15	23	30	56	56	85	98	92	104	84	76	67	59	49	27	24	9	9	5	5	2	3	2			1			
	∪	963	2	1	3	4	5	4	14	8	13	38	50	74	67	83	102	113	89	81	60	47	35	28	25	21	7	10	3	4	2	2		1	2			1
	0·0	535	4	2	4	5	9	4	15	6	15	41	54	52	71	97	92	129	118	62	62	35	43	32	21	11	7	5	2						2			
	0·1	1084			1	3	1	4	11	14	17	32	51	74	76	85	100	99	72	87	64	60	40	26	27	18	8	9	5	5	4	3		1	2			
ZE	*y*	771	1	3	3	10	18	18	38	25	31	60	40	49	84	69	52	99	51	47	62	27	32	43	26	22	21	8	17	4	5	8	6	1	3	3	3	
	s	265					7	4	11	23	30	45	41	49	113	94	79	64	68	64	57	38	38	45	34	30	19	8	19			8	4				4	
	e	232		4		9	13	13	13	22	9	52	43	47	77	90	82	99	90	43	56	39	26	39	22	17	26	17	5	4	13		9	4		9		
	w	274	4	4	7	22	33	36	84	29	51	80	36	51	62	26	0	131	0	33	73	7	33	44	22	18	18	0	25	7	4	14	7		7		4	
ZG	*y*	788	3		3	5	3	8	25	28	27	53	46	70	55	80	72	85	61	63	48	68	48	27	33	23	15	13	10	8	5	4	1	4		1	2	1
	s	272					7	7	19	26	33	29	44	52	48	77	110	88	63	92	74	44	44	29	29	18	18	15	7		11	4		4		4	4	
	e	231	4			4		5	17	17	22	56	30	74	61	69	87	69	104	56	39	82	69	30	26	35	9	9	5	13		4		4				
	w	285	3		7	10		10	39	39	25	74	60	84	56	91	25	95	25	42	32	81	35	21	42	18	18	14	18	10	4	4	3	3			3	3
E + G......	∩	644	3	2	3	12	11	25	25	26	33	43	45	59	59	76	71	109	50	50	47	51	48	26	28	22	23	12	14	6	3	5			3		3	2
	∪	915	1	1	2	4	10	4	36	26	26	66	42	60	77	73	56	80	60	59	61	46	35	40	31	23	14	9	13	6	7	7	7	4		3	2	
	0·0	515			4	13	15	12	33	37	27	60	52	47	66	83	70	109	47	60	56	47	35	41	25	23	10	8	2		6	2	4	2	2	2		
	0·1	1044	3	2	2	5	8	18	30	21	30	55	38	66	71	70	58	83	60	53	54	49	43	32	31	22	22	11	19	9	5	8	4	3	1	2	4	1

of the table and figure shows the following main features: (*a*) the positive departures extend to more extreme values (including some, over 100, not

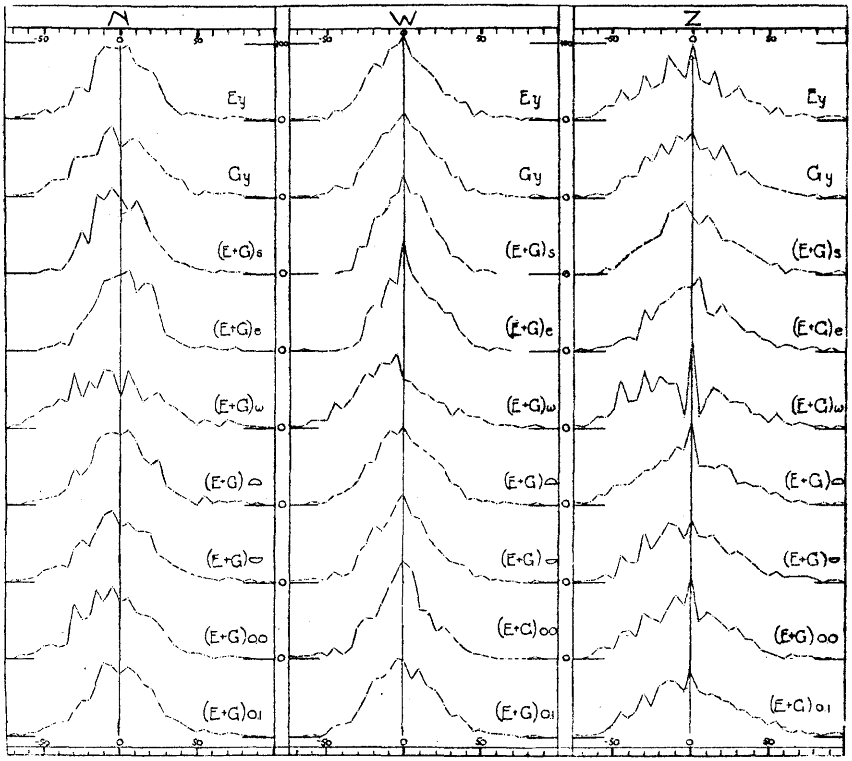

Fig. 3.—Frequency Curves for ΔR. The Vertical Interval between the base lines of successive curves represents 100 units.

shown in fig. 3) than the negative, and this difference has to be balanced by an excess of moderate negative departures; but the considerable degree of symmetry in the distribution of positive and negative departures is more remarkable than the slight asymmetry; (*b*) the distribution of ΔR in each element is approximately the same at G as at E, hence in fig. 3 only the *y* curves are given for the two stations separately; (*c*) the spread of ΔR is least in summer and greatest in winter; since, however, ΔR is a percentage of the mean range, which is much less in winter than in summer, the spread of the actual departures, $R - R_n$, from the normal values, is least in winter*; (*d*)

* The jagged appearance of the curves for Z in fig. 3, especially for *w*, arises because R is measured only to 1γ, and a change of R by 1 γ may alter the value of ΔR by 10 per cent., since the mean range in Z is of the order 10 to 14 γ in *w*. Since Table V relates to 5 per cent. intervals of ΔR, the frequencies alter rather abruptly in successive columns; they and the corresponding curves should be smoothed by taking overlapping means, to get a fair representation of the distribution of ΔR.

there is but little systematic difference between the percentage departures ΔR in $\sqcap$ and $\sqcup$ years ; (*e*) the spread of ΔR on 0·0 and 0·1 days is nearly the same, though large positive values are more common for 0·1 than for 0·0 days.

These conclusions are more precisely illustrated by the mean values of the positive and negative departures separately, denoted by $|\Delta_+\mathrm{R}|$, $|\Delta_-\mathrm{R}|$, and by the mean departure $|\Delta\mathrm{R}|$ having no regard to sign ; these are given in Table VI. The mean values $|\Delta\mathrm{R}|$ are smaller than the means of $|\Delta_+\mathrm{R}|$ and $|\Delta_-\mathrm{R}|$, because there were days of zero ΔR.

Table VI.—Mean Positive, Negative and Arithmetical Departures of Ranges from their Standard Ranges.

Observatory Station.	Year, season or day.	N.			W.			Z.		
		$\|\Delta_+\mathrm{R}.\|$	$\|\Delta_-\mathrm{R}.\|$	$\|\Delta\mathrm{R}.\|$	$\|\Delta_+\mathrm{R}.\|$	$\|\Delta_-\mathrm{R}.\|$	$\|\Delta\mathrm{R}.\|$	$\|\Delta_+\mathrm{R}.\|$	$\|\Delta_-\mathrm{R}.\|$	$\|\Delta\mathrm{R}.\|$
E	*y*	19·1	18·2	17·7	19·3	17·7	17·4	30·9	25·4	25·1
	s	16·4	12·6	13·5	15·0	12·9	13·6	25·5	18·9	20·6
	e	16·4	14·0	14·6	17·9	13·8	15·1	27·8	21·1	21·9
	w	24·9	26·0	24·4	27·0	23·9	23·0	40·8	34·6	32·2
G	*y*	22·5	20·2	20·3	17·7	16·9	16·5	25·9	22·0	21·8
	s	20·0	16·9	17·6	14·2	13·6	13·6	23·8	19·6	19·7
	e	19·9	16·9	17·2	16·9	12·9	14·2	22·3	20·0	19·7
	w	27·9	25·0	25·4	23·6	21·4	21·2	31·9	25·3	25·6
E + G	$\sqcap$	19·3	17·2	17·3	17·7	16·6	16·5	27·4	24·2	22·8
	$\sqcup$	21·9	20·5	20·2	19·0	17·8	17·3	28·8	23·4	23·9
	0·0	19·9	20·7	19·4	16·1	18·2	16·1	23·7	24·3	21·4
	0·1	21·2	18·3	18·8	19·6	16·9	17·4	30·1	23·4	24·5

Examination of the Type of S_q *on Days of Widely Different Range.*

§ 15. In order to determine whether the type of S_q at any epoch varied with the range, it was judged sufficient to consider the hourly inequalities for three groups of days of different range at one station (E) and one season only. Actually the examination was confined to the 156 quiet days of June and July, when S_q has its largest amplitude. For each element these days were divided into three nearly equal groups according to their values of ΔR for that element ; the mean diurnal hourly inequality was then formed for each group. The number of days, and the mean range, for each such group are shown in Table VII, which also gives the average hourly numerical departure of the element from its mean value, derived from the inequality for the group.

Table VII.

Element.	Group of days.	No. of days.	Mean R.	Average departure.	Mean R ÷ average departure.
N	Large Δ_{-}R	51	38·6	10·0	3·9
	Small ΔR	50	45·1	11·8	3·8
	Large Δ_{+}R	53	53·3	13·5	3·9
W	Large Δ_{-}R	51	42·8	11·0	3·9
	Small ΔR	48	54·4	13·5	4·0
	Large Δ_{+}R	53	64·9	14·8	4·4
Z	Large Δ_{-}R	47	13·2	3·1	4·3
	Small ΔR	47	18·9	4·2	4·5
	Large Δ_{+}R	46	26·1	6·0	4·4

The inequalities or daily variations are plotted in fig. 4, for each element; the curves show that the considerable differences of amplitude in the diurnal

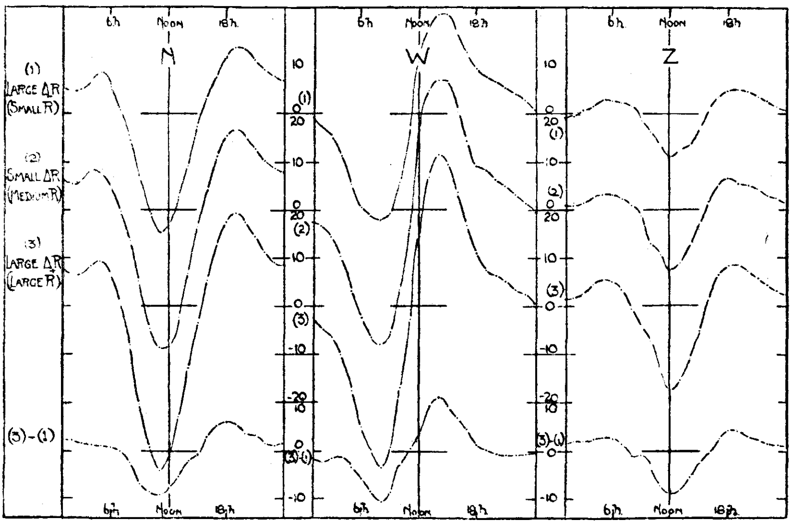

FIG. 4.—DIURNAL VARIATIONS ON QUIET DAYS, GROUPED ACCORDING TO THEIR ΔR. (Eskdalemuir, June–July, 1913–23.)

inequalities for the three groups are unaccompanied by any appreciable change of type; this is more clearly brought out by the lowest curves, which represent the difference between the curves for the groups of largest and smallest range. These curves are almost exactly of the same type as the others, showing that the curves of largest amplitude are merely magnified versions of the curves of smaller amplitude. The same fact is illustrated in another way by the similarity of the ratios of the mean ranges to the average departures for the inequalities for the three groups of days for each element, as indicated in the last column of Table VII. The evidence here afforded is taken to be sufficient proof of the adequacy of R as an index of the S_q variation at any epoch.

The Relation between Corresponding Values of ΔR *for the same Element at the Two Observatories.*

§ 16. It has been seen that the distribution of the fortuitous changes in R is much the same for all three elements and at the two observatories. It is important to ascertain how far the percentage departures of any element at one observatory are paralleled by those of the same element at the other observatory.

To examine this point, the mean values of ΔR (NE) and ΔR (NG) were formed for the mean values of the groups of days for which ΔR (NE), at any season or for the whole year, lay within the successive intervals 5 units wide considered in Table V: the value of ΔR (NG) on any day did not always lie in the same interval as ΔR (NE) for that day. Sets of pairs of corresponding mean values of ΔR (NE) and ΔR (NG) were thus obtained, for the three seasons and the whole year; these pairs of values were taken as co-ordinates of points on four curves, which are reproduced in fig. 5 (*a*), as curves (1)–(4), in the order *s*, *e*, *w* and *y*. Alongside each point is given the number of days from which the corresponding mean values of ΔR were derived; points based on less than 5 days are not included in the diagram.

Curves (5)–(8) and (9)–(12) were constructed, similarly, from the values of ΔR (W) and ΔR (Z) at E and G, for groups of days at which ΔR (WE) or ΔR (ZE) fell within the 5-unit intervals of Table V.

If each element always changed in the same proportion at the two observatories, the curves in fig. 5 (*a*) would all be straight lines inclined at 45° to the axes, corresponding to ΔR (E) $= \Delta$R (G). The figure shows that for N and W the curves approximate to such straight lines, though with accidental irregularities; that is, these elements do vary nearly proportionately at the two stations. The agreement is closest in summer. On the other hand, for Z the curves depart somewhat widely from the form ΔR (E) $= \Delta$R(G), and are much more irregular than for N and W.

These facts are summed up in a brief and quantitative manner by the following correlation coefficients between ΔR(E) and ΔR(G); these coefficients have been calculated directly from the whole set of individual values of ΔR.

Table VIII.—Correlation Coefficients.

—	ΔR (NE), ΔR (NG).	ΔR (WE), ΔR (WG).	ΔR (ZE), ΔR (ZG).
Summer	0·798	0·871	0·642
Equinox	0·761	0·847	0·504
Winter	0·761	0·801	0·253
Year	0·773	0·840	0·466

The table shows that W is the element in which the departures from the normal range are most clearly correlated at the two observatories, while in Z the relation is least close, though even for Z the correlation in summer reaches a high value. These high correlations incidentally testify to the accuracy of the data afforded, quite independently, by the two observatories; if the data were affected by much accidental error, such high correlations could hardly be manifested.

The Relation between Corresponding Values of ΔR *for Different Elements at the same Observatory.*

§ 17. The correlation between the proportionate departures of the different elements from their normal values at the same observatory was next examined, on lines similar to those of § 16. In this section of the discussion the E and G data were combined without distinction. In considering the relations between N and W, and N and Z, the mean values of ΔR were determined for the groups of days on which ΔR (N) lay within the 5-unit intervals of Table V ; in considering the relation between W and Z, the groups of days used were those for which ΔR (W) lay within such intervals. The results, in the form of graphs in which the co-ordinates are ΔR (N), ΔR (W), or ΔR (N), ΔR (Z), or ΔR (W), ΔR (Z), are plotted in fig. 5 (*b*), in three sets each of four curves, for the seasons *s*, *e*, *w* and for the year. If the fortuitous variations of S_q affected the three elements proportionately, all these curves would be straight lines inclined at 45° to the axes, but actually they depart from this ideal far more than do the curves in fig. 5 (*a*). The relation between N and W seems to be the least affected by accidental irregularities, while that between N and Z seems most affected.

The following correlation coefficients illustrate the same facts in a more definite but less detailed way. The E and G data have here been separately considered, and also those for the ◠ and ◡ groups of years. In the determination of each coefficient neither element involved has been treated as primary, in the sense that N (or W) was taken as primary in forming the graphs of fig. 5 (*b*).

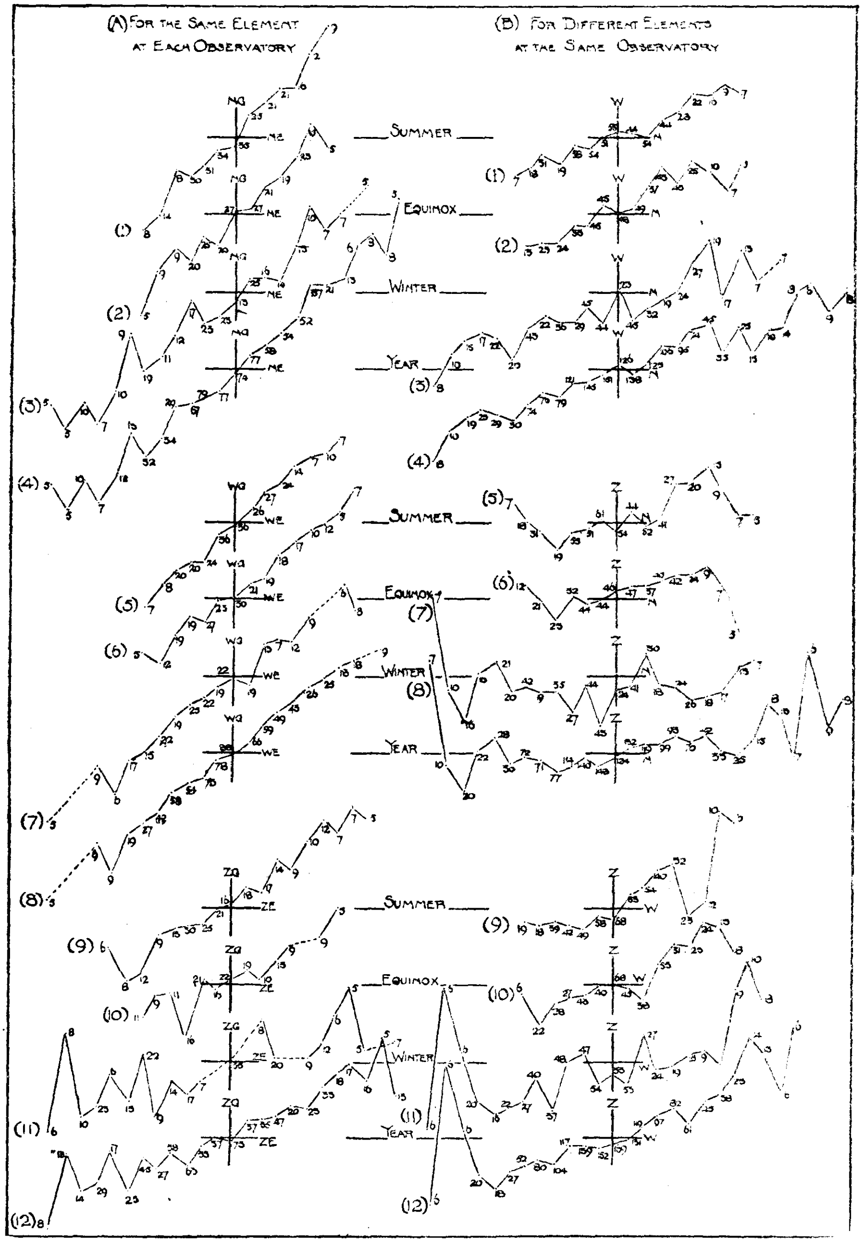

Fig. 5.—Corresponding Pairs of Mean Values of ΔR. Length of axis from origin represents 10 per cent. ΔR in each diagram.

Table IX.—Correlation Coefficients.

——	ΔR (NE), ΔR (WE).	ΔR (NE), ΔR (ZE).	ΔR (WE), ΔR (ZE).	ΔR (NG), ΔR (WG).	ΔR (NG), ΔR (ZG).	ΔR (WG), ΔR (ZG).
Summer	0·340	0·249	0·304	0·405	0·183	0·240
Equinox	0·466	0·125	0·333	0·424	0·050	0·290
Winter	0·329	0·156	0·339	0·414	−0·014	0·169
Year	0·378	0·177	0·325	0·414	0·073	0·233
$\sqcap$	0·399	0·233	0·379	0·438	0·052	0·264
$\sqcup$	0·370	0·115	0·306	0·420	0·052	0·147

The most striking feature of the above table is the much smaller correlations throughout, as compared with those shown by Table VIII. Another curious feature is that the N–Z and W–Z coefficients are smaller for G than for E, although for N–W the relationship is closer at G than at E. Summer is no longer the season in which all the relationships are closest, as was the case in Table VIII. The correlations appear somewhat closer in $\sqcap$ than in $\sqcup$ years.

Interpretation of the S_q *Variability.*

§ 18. Though it is not possible, as yet, to interpret the results of §§ 11–17 in any detail, it may be of interest to consider briefly their significance in relation to the overhead electric current system which is the main cause of S_q. This current-system is of simple form, consisting of four families of oval curves, two north and two south of the equator, and separated approximately by the sunrise and sunset meridians ; the current-intensity is much greater over the day than over the night hemisphere. The current system is stationary in form relative to the meridian plane through the sun ; the rotation of the earth carries any station round a circle of latitude drawn in this spherical current-system, and hence S_q depends on local time, being repeated at corresponding local times at different stations in the same latitude.

The S_q variations in N and W are determined mainly by the (intensity and direction of the) currents within a not very large distance of each station, at each time of day ; the Z variation depends rather on the rate of space-variation of the overhead currents. An increase in the amplitude of S_q betokens a general intensification of the current-system ; the constancy of the type of S_q in any element, throughout the changes of range (§ 15), indicates that the general form of the current-system, and especially the great difference between the day and night hemispheres, is preserved while the current intensity changes. The fact that the S_q ranges for the different elements at the same station do not always change in the same proportion indicates, however, that the current-system does not remain absolutely constant in form; the direction of the current-flow undergoes changes, thus altering the ratio of R (N) and R (W), while the space-variation of the currents is also modified, affecting R (Z). But the current system is so extensive and so simple in form that any such changes in it affect large areas in a similar manner; hence we see that the changes in any one element at E and G are closely correlated.

§ 19. The preceding section completes the main part of this paper. Having shown the existence, amount, and principal properties of the irregular changes of S_q at two stations not very far apart, the next step must be to determine how far the variations are worldwide, that is, what degree of correlation is shown between the values of ΔR for more distant stations. If the correlation is found to be considerable, it is hoped to develop a practical plan for the assignment to each Greenwich day of an index of the intensity of S_q on that day, whether, as on quiet days, S_q appears in its pure form, or whether, as during periods of magnetic activity, S_q is overlaid by irregular magnetic changes. Such daily indices should be of value in the investigation of solar and terrestrial relationships, since they relate to a feature of the magnetic variations quite different from that to which the daily magnetic character figures refer. It is intended to compare these S_q indices with various solar and geophysical data, such as the ozone content of the atmosphere, the sun's ultra-violet radiation, and radio data.*

At the present stage it is premature to assign daily indices even for the quiet days here used. But some interesting and unexpected correlations of a local kind, which may, however, have a wider significance, have been found, between ΔR and α and D (§ 2). This part of the work has been confined to the E data in view of the close correlation found between the E and G results; similar results may be expected to hold good for the G data also.

§ 20. The values of ΔR for each element (and for each season) were divided into three nearly equal groups corresponding to days of large negative departures, small departures, and large positive departures from the normal ranges. The values of α and D for these groups of days were tabulated and their means determined. In the case of N a rather more elaborate process was adopted: "normal" values of α (N) and D (N) were determined by a procedure similar to that used in the case of R_n, and the tabulations gave $\alpha - \alpha_n$, $D - D_n$ instead of the simple values of α and D, as for the other elements.

The results are given in the following Tables X, XI, and XII. The three seasons and the year were separately treated, and the $\sqcap$ and $\sqcup$ groups of years were also considered separately and together.

* *Cf.* Chapman, 'Nature,' vol. 121, p. 989 (1928).

Table X.—Mean Values of $\alpha - \alpha_n$, $D - D_n$ for NE for sets of Days grouped according to their values of ΔR (NE).

	Days of large negative ΔR.				Days of small ΔR.				Days of large positive ΔR.				Means.	
	Number of days.	Mean ΔR.	Mean $\alpha-\alpha_n$.	Mean $D-D_n$.	Number of days.	Mean ΔR.	Mean $\alpha-\alpha_n$.	Mean $D-D_n$.	Number of days.	Mean ΔR.	Mean $\alpha-\alpha_n$.	Mean $D-D_n$.	α.	D.
All years	264	−23·0	−0·64	0·95	272	−1·6	0·08	−0·10	270	24·0	0·23	−0·86	2·40	2·37
∩	102	−21·6	−0·91	1·01	114	−0·5	0·59	0·03	109	22·4	−0·28	−0·91	3·46	3·20
∪	162	−23·9	−0·48	0·91	158	−2·5	−0·28	−0·19	161	25·1	0·57	−0·83	1·68	1·81
s	93	−16·4	−0·70	0·76	92	−1·0	−0·03	−0·15	94	19·8	0·17	−0·51	2·56	0·90
s ∩	39	−16·7	−1·13	1·36	39	−0·9	0·87	−0·03	37	16·8	−1·68	0·00	3·39	1·51
s ∪	54	−16·1	−0·39	0·33	53	−1·0	−0·70	−0·25	57	21·7	1·37	−0·84	1·98	0·48
e	76	−16·5	−0·68	0·97	83	+2·0	−0·36	0·20	81	22·5	0·49	−1·10	2·65	3·18
e ∩	29	−15·1	−1·14	0·03	35	+3·0	−0·26	−0·46	38	21·0	1·34	−1·55	4·11	3·52
e ∪	47	−17·4	−0·40	1·55	48	+1·3	−0·44	0·69	43	23·8	−0·26	−0·70	1·57	2·93
w	95	−34·8	−0·56	1·11	97	−5·4	0·57	−0·31	95	29·5	0·06	−1·00	2·03	3·12
w ∩	34	−32·8	−0·47	1·44	40	−3·2	1·05	0·50	34	30·1	−0·56	−1·18	2·94	4·69
w ∪	61	−35·9	−0·61	0·92	57	−7·0	0·23	−0·88	61	29·2	0·41	−0·90	1·48	2·17

Table XI.—Mean Values of α (WE) and D (WE) for sets of Days grouped according to their Values of ΔR (WE).

	Days of large negative ΔR.				Days of small ΔR.				Days of large positive ΔR.				Mean.	
	Number of days.	Mean ΔR.	Mean α.	Mean D.	Number of days.	Mean ΔR.	Mean α.	Mean D.	Number of days.	Mean ΔR.	Mean α.	Mean D.	α.	D.
All years	275	−22·2	1·72	0·76	267	−1·8	1·19	0·89	264	23·7	0·98	0·93	1·30	0·86
∩	109	−21·7	2·38	1·15	108	−1·7	1·18	1·47	108	23·5	1·16	1·40	1·57	1·34
∪	166	−22·6	1·28	0·50	159	−1·8	1·19	0·50	156	23·9	0·87	0·60	1·12	0·53
s	94	−16·2	1·07	0·13	93	0·7	1·11	0·42	91	19·7	0·86	0·28	1·01	0·28
s ∩	33	−16·3	1·15	0·70	39	2·0	1·13	0·85	42	20·1	0·71	0·48	0·98	0·67
s ∪	61	−16·1	1·04	−0·18	54	−0·4	1·09	0·11	49	19·4	0·98	0·10	1·04	0·01
e	82	−16·3	2·94	1·02	77	2·2	0·79	1·47	80	24·0	0·69	2·03	1·49	1·50
e ∩	34	−14·6	3·41	1·50	32	1·7	0·25	1·56	36	23·5	1·39	2·56	1·71	1·89
e ∪	48	−17·5	2·60	0·69	45	2·8	1·18	1·40	44	24·5	0·11	1·59	1·34	1·21
w	99	−32·9	1·31	1·13	97	−7·3	1·58	0·89	93	27·4	1·37	0·62	1·42	0·88
w ∩	42	−31·7	2·50	1·21	37	−8·6	2·03	2·05	30	28·2	1·50	1·30	2·06	1·52
w ∪	57	−33·8	0·44	1·07	60	−6·5	1·30	0·17	63	27·0	1·30	0·30	1·03	0·50

Table XII.—Mean Values of α (ZE) and D (ZE) for sets of Days grouped according to their values of ΔR (ZE).

	Days of large negative ΔR.				Days of small ΔR.				Days of large positive ΔR.				Mean.	
	Number of days.	Mean ΔR.	Mean α.	Mean D.	Number of days.	Mean ΔR.	Mean α.	Mean D.	Number of days.	Mean ΔR.	Mean α.	Mean D.	α.	D.
All years	252	—31·3	—1·07	0·40	259	—5·1	—0·69	—0·13	257	34·9	0·33	0·58	—0·48	0·28
$\cap$	104	—32·9	—1·35	0·52	119	—5·5	—0·88	0·30	106	33·7	0·25	1·59	—0·66	0·79
$\cup$	148	—30·2	—0·88	0·31	140	—4·7	—0·54	—0·49	151	35·7	0·38	—0·13	—0·34	—0·10
s	90	—23·6	—0·50	0·91	88	—1·1	0·02	1·40	87	30·9	0·86	0·28	0·12	0·86
s $\cap$	32	—22·3	—0·78	0·72	38	—1·8	—0·08	2·39	46	31·0	1·28	0·98	0·27	1·37
s $\cup$	58	—24·3	—0·34	1·02	50	—0·5	0·10	0·64	41	30·8	0·39	—0·51	0·01	0·47
e	72	—27·2	—0·96	0·26	78	—2·2	—0·50	—1·12	79	32·6	0·09	1·57	—0·44	0·24
e $\cap$	26	—30·0	—0·73	—0·04	43	—2·5	—0·91	—0·84	33	34·6	—0·91	3·18	—0·85	0·67
e $\cup$	46	—25·6	—1·09	0·43	35	—1·9	0·00	—1·46	46	31·1	0·80	0·41	—0·10	—0·09
w	90	—42·3	—1·73	—0·01	93	—11·2	—1·54	—0·74	91	40·6	0·02	0·01	—1·08	—0·25
w $\cap$	46	—42·0	—2·09	0·70	38	—12·4	—1·66	—0·50	27	37·4	—0·07	0·70	—1·45	0·29
w $\cup$	44	—42·6	—1·36	—0·75	55	—10·4	—1·45	—0·91	64	42·0	0·06	—0·28	—0·83	—0·62

Discussion of Tables X, XI, XII.

§ 21. The foregoing tables show that the mean values of α and D in N and W are positive, while in Z they are smaller, and perhaps change sign in the course of the year, the mean values being negative for α and positive for D. In all three elements, but especially in Z, the mean values are small, of the order 1γ, so that they are liable to a proportionately larger accidental error than R.

The positive mean values of α and D for N and W correspond to the known fact that since disturbance reduces the mean value of H, on quiet days the mean value is above the mean H for all days, and H is on the whole increasing, by way of recovery from the disturbance depression. Since H at E and G has N and W components, both positive, α and D for N and W are positive.

Disturbance tends to raise the mean value of Z, but only very slightly; a similar argument would imply that α and D for Z should be negative; this is true for α, in the mean of the year, while D appears to be positive, but so small that its sign is perhaps doubtful.

The novel feature of Tables X to XII, however, is the evaluation of α and D for quiet days of different S_q range; the principal results which appear are :—

(*a*) There is a considerable and systematic decrease of D (N) as R (N) increases.
(*b*) There is a distinct and fairly regular increase of α (N) with R (N).
(*c*) There is no marked regular change of α and D with R in W.
(*d*) There is no marked regular change of D (Z) with R (Z).
(*e*) There is a considerable and systematic (algebraic) increase in α (Z) with R (Z).

The relations (*a*), (*b*) point to an inverse correlation between α (N) and D (N) on quiet days; this appears not to have been noticed hitherto. Such a relation is to be expected on the days (whether quiet or not) following a large magnetic disturbance, for then D is negative and numerically decreasing, but algebraically increasing, while α, the rate of recovery towards normal, decreases as the normal is approached. If the normal or undisturbed value of H is not attained, even after many days, this relation should be maintained even when D is positive, the real correlation, however, being that between α and the departure from the undisturbed (not the mean) value. But this explanation gives no account of the correlation of α and D with the S_q range R. This correlation is an unexpected one precisely because α and the changes in D are usually thought of as connected with magnetic disturbance, whereas it has seemed doubtful whether there is any systematic connection between disturbance and the S_q range. No explanation of the relationships (*a*)-(*e*) is offered, but they seem likely to have an interesting significance, and to be worthy of further study in the records of other observatories.

Summary.

§ 22. The regular changes of the solar diurnal magnetic variation on really quiet days at Eskdalemuir and Greenwich, in the course of the year and the sunspot cycle, are investigated, and used to define a "normal" range R_n of the daily variation on each such quiet day in the period 1913–23. The actual ranges R are found to differ from the normal ones R_n, and their percentage departures (ΔR) from the normals are investigated. The average numerical departures ΔR for different elements and seasons range from about 20 to 30 per cent.; the distribution is fairly symmetrical about the mean, and similar in different elements and for the two observatories. It is found that corresponding daily values of ΔR for the same element at the two observatories are closely correlated, whereas there is much less correlation between corresponding values of ΔR for different elements at the same observatory. It is shown that R or ΔR sufficiently characterises the daily variation at any season, because the variation is the same, except in scale, on days of large as on days of small range. Finally, some distinct and unexpected relationships are found between the values of ΔR, the non-cyclic variation, and the departure of the daily mean from the monthly mean values of the horizontal and vertical magnetic force.